Performance of Mobile Telecommunications Network with Overlapping Location Area Configuration

Shun-Ren Yang, Yung-Chun Lin and Yi-Bing Lin*, *Fellow, IEEE*

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Abstract

In a mobile telecommunications network, the location area (LA) of a mobile station (MS) is tracked by the location area update (LAU) mechanism. To reduce the LAU traffic caused by the ping-pong effect, the overlapping LA concept is introduced. In the overlapping LA configuration, an LA selection policy is required to select the new LA at an LAU when the MS enters a new cell covered by multiple LAs. This paper describes four LA selection policies and proposes an analytic model to study the performance of these LA selection policies. Our study provides guidelines to determine appropriate degree of overlapping among the LAs.

Key words: location area, location update, mobility management, ping-pong effect, overlapping LA

1 Introduction

In a mobile telecommunications network, the cells (the coverage areas of base stations) are partitioned into groups. These groups are referred to as the *location areas* (LAs; in GSM [7] and in the CS domain in GPRS/UMTS [8]), the *routing areas* (RAs; in the PS domain in GPRS/UMTS), the *paging groups* (PGs; in WiMax [6]), and so on. Without loss of

*Corresponding Author: Yi-Bing Lin, Dept. Comp. Sci. & Info. Engr., National Chiao Tung University, Hsinchu, Taiwan, R.O.C.; Fax:+886-3.5724176; Email: liny@csie.nctu.edu.tw

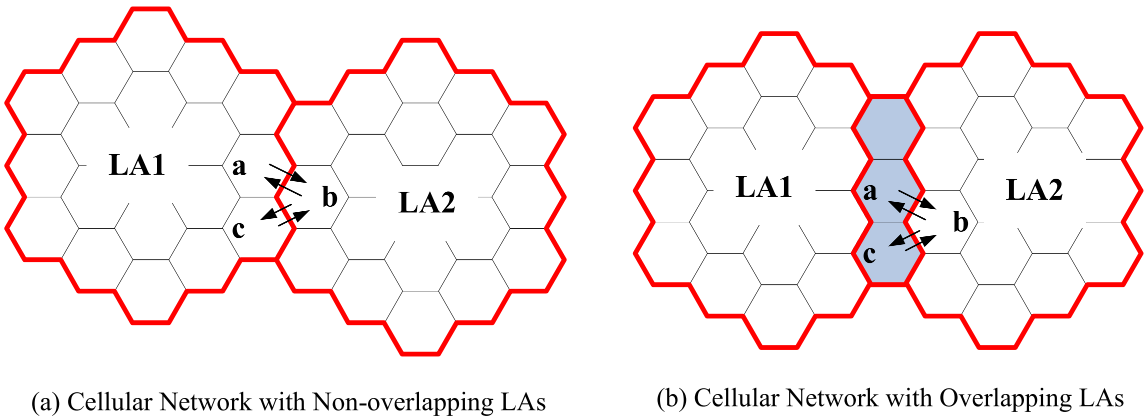

Figure 1: Ping-pong Effect Reduction with Overlapping LA Configuration

generality, we use the term LA throughout this paper. The LA of a *mobile station* (MS) is tracked by the network. Whenever the MS moves from one LA to another LA, the MS issues a location area update (LAU) request to inform the network that it has entered the new LA. In this way, the network keeps track of the LA where the MS resides.

A typical LA layout is illustrated in Figure 1 (a) where every cell is covered by exactly one LA. Recently, the overlapping LA concept (where a cell is covered by multiple LAs; see the gray areas in Figure 1 (b)) has been proposed to avoid the *ping-pong effect* [1, 2, 3, 4, 5, 9]. Suppose that an MS resides in cell a of LA 1 as illustrated in Figure 1. In the non-overlapping LA configuration (Figure 1 (a)), when the MS moves to cell b in LA 2, the MS performs an LAU to register to LA 2. If the MS moves to cell c or back to cell a in LA 1 again, the MS performs another LAU to register to LA 1. If the MS repeatedly moves back and forth between LA 1 and LA 2, many LAUs are performed. This phenomenon is called the ping-pong effect. In the overlapping LA configuration, when the MS moves from cell a (in LA 1) to cell b (in LA 2), an LAU is executed as in the non-overlapping LA configuration. If the MS moves to cell c or back to cell a, it still resides in LA 2 and no LAU is performed. Therefore, the ping-pong effect is mitigated.

Several studies [1, 2, 5] proposed analytic models to study the performance of the mobile telecommunications network with overlapping LAs. These studies assumed that at each movement, the MS moves to any of its neighboring cells with the same probability. In [5], the LA consists of odd numbers of cells. In [1, 2], an example is provided and no close-form solution was derived to evaluate the performance. Moreover, each of the previous studies investigated one LA selection algorithm. In this paper, we propose an analytic model to investigate the performance for four LA selection policies where the MS can move to each of its neighboring cells with different probabilities.

This paper is organized as follows. Section 2 introduces four overlapping LA policies and the corresponding analytic models. Section 3 quantitatively compares the four studied policies. Then Section 4 provides guidelines to determine appropriate degree of overlapping among the LAs.

2 Analytic Model for Overlapping Location Area Configuration

This section proposes an analytic model to study the LAU costs for mobile telecommunications networks with overlapping LAs. For the purpose of demonstration, one-dimensional overlapping LA configuration is considered. Based on this configuration, we describe a random walk model for MS movement. Then we construct state transition diagrams to derive the LAU costs.

Figures 2 and 3 illustrate the one-dimensional overlapping LA configuration, where each LA covers N cells, and is overlapped with each of its adjacent LAs by K cells ($0 \leq K < N$). We say that the *overlapping degree* for this LA configuration is K. In each LA, cells are sequentially labelled from 1 to N. An MS moves to the right-hand side neighboring cell with the *routing probability* p, and moves to the left-hand side neighboring cell with probability $1 - p$. If the MS moves into a new cell that does not belong to the currently registered LA (i.e., the MS moves out of the current LA), it performs an LAU. This new cell is called the *entrance cell* to the new LA. If $0 \leq K < \frac{N}{2}$, the entrance cell is covered by only one LA. When the MS moves out of LA i from the right-hand side (the left-hand side), it enters LA

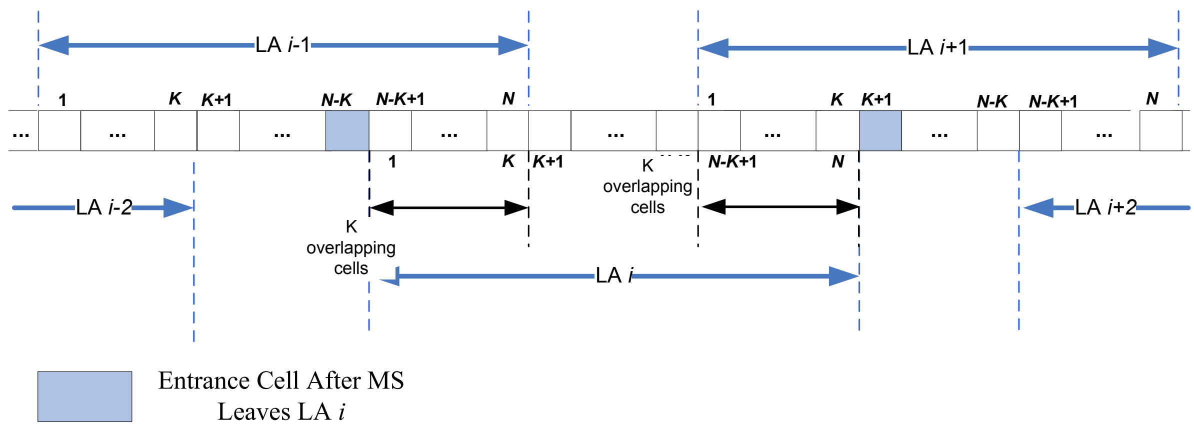

Figure 2: One-dimensional Overlapping LA Registration Scheme for $0 \leq K < \frac{N}{2}$

$i+1$ (LA $i-1$). The entrance cell is cell $K+1$ of LA $i+1$ (cell $N-K$ of LA $i-1$); see the shadow boxes in Figure 2. On the other hand, if $\frac{N}{2} \leq K < N$, when an MS leaves the old LA and enters the new cell, this new cell is covered by several LAs adjacent to (or overlapping with) the old LA. As shown in Figure 3, when the MS leaves LA i, the new cell can be the entrance cell for each of N_O LAs, where

$$N_O = \left\lceil \frac{K+1}{N-K} \right\rceil . \tag{1}$$

That is, if the MS moves out of LA i and enters a right-hand side LA, the entrance cell can be cell $K+1$ of LA $i+1$, cell $N+1-2(N-K)$ of LA $i+2$, $N+1-3(N-K)$ of LA $i+3$, $\cdots$, or cell $N+1-N_O(N-K)$ of LA $i+N_O$ (see the shadow boxes in Figure 3). Similarly, if the MS moves out of LA i and enters a left-hand side LA, the entrance cell can be cell $N-K$ of LA $i-1$, cell $2(N-K)$ of LA $i-2$, $\cdots$, or cell $N_O(N-K)$ of LA $i-N_O$. Since the new cell is covered by more than one LAs for $\frac{N}{2} \leq K < N$, a policy is required to select the new LA at an LAU. In this paper, we investigate four LA selection policies described as follows.

- In the *Maximum Overlapping* (MaxOL) policy [1, 2, 5], after moving out of the current LA i from the right-hand side (the left-hand side), the MS will register to the adjacent LA $i+1$ (LA $i-1$). In this case, the number of cells overlapped between the old and

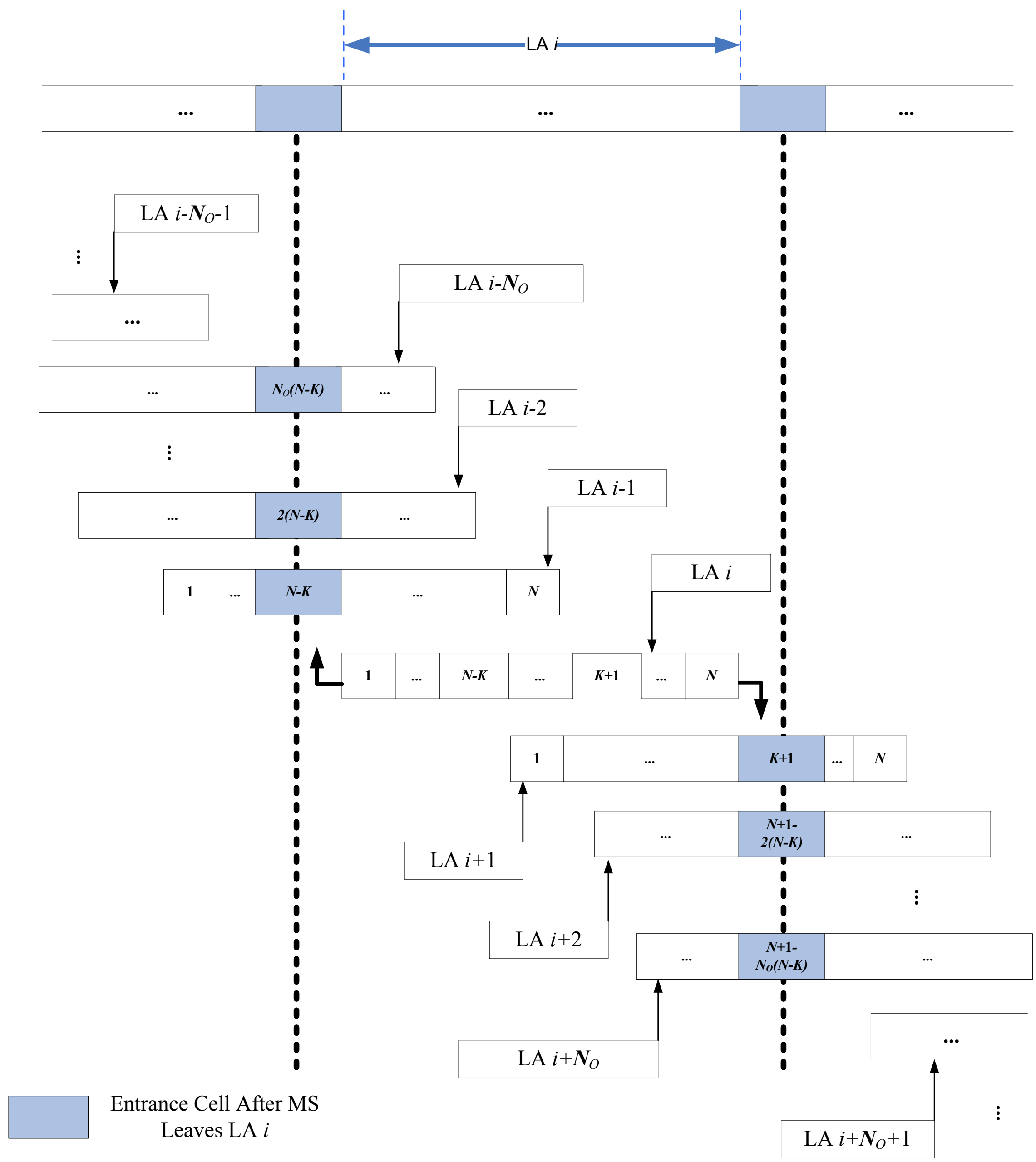

Figure 3: One-dimensional Overlapping LA Registration Scheme for $\frac{N}{2} \leq K < N$

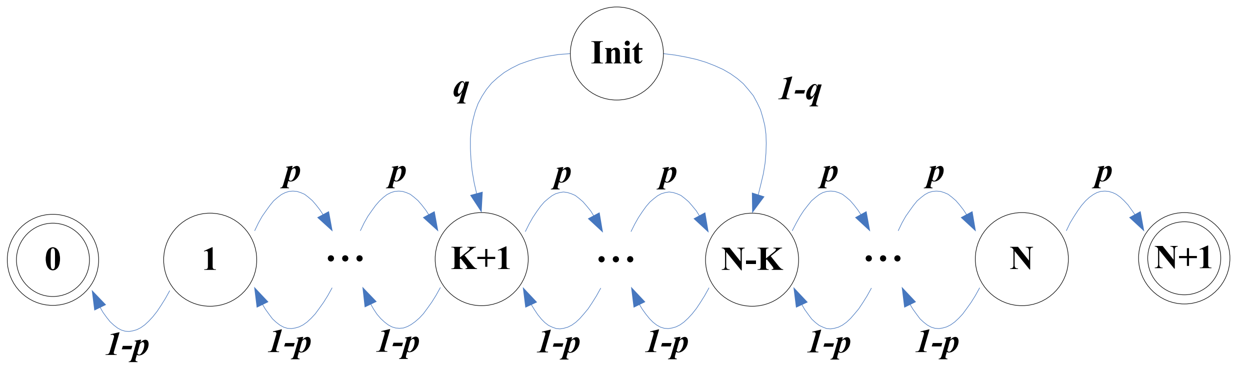

Figure 4: State Transition Diagram for K-degree Overlapping LA Configuration (the LA Size is N, and $0 \leq K < \frac{N}{2}$)

the new LAs is maximal.

- In the *Central* policy [5], after moving out of the current LA, the MS always registers to the LA whose central cell is closest to the entrance cell.
- In the *Random* policy, the MS randomly registers to one LA covering the entrance cell.
- In the *Minimum Overlapping* (MinOL) policy, after moving out of LA i, the MS chooses the farthest LA of the entrance cell from LA i (i.e., LA $i + N_O$ in the right-hand side and LA $i - N_O$ in the left-hand side in Figure 3). In this case, the number of cells overlapped between the old and the new LAs is minimal.

For $0 \leq K < \frac{N}{2}$, the above four policies are the same; that is, the only LA covering the entrance cell is selected. Suppose that an MS makes M cell movement before it leaves an LA. For each of the above policies, we derive the expected number $E[M]$. It is clear that the larger the $E[M]$ value, the better the performance.

2.1 Case 1: $0 \leq K < \frac{N}{2}$

Figure 4 illustrates the state transition diagram for MS cell movement in an LA, where $0 \leq K < \frac{N}{2}$. In this diagram, state *Init* represents that the MS moves into the LA in the steady state. State j represents that the MS resides in cell j of the LA, where $1 \leq j \leq N$.

Two virtual states, 0 and $N+1$, are the absorbing states representing that the MS moves out of the LA from cell 1 and from cell N, respectively. For $1 \le j \le N$, the MS moves from state j to state $j+1$ with probability p, and the MS moves from state j to state $j-1$ with probability $1-p$. As mentioned before, the entrance cell can be cell $K+1$ (cell $N-K$) of the new LA when the MS leaves the old LA from the right-hand side (the left-hand side). Let q be the probability that the MS moves from the old LA to the new LA through the entrance cell $K+1$. Then the MS moves from state $Init$ to state $K+1$ with probability q and to state $N-K$ with probability $1-q$. Note that q is affected by the routing probability p, the overlapping degree K and the LA size N. We will derive q later.

Starting from the entrance cell j, let N_j be the number of cell movement before the MS leaves the LA. The expected number $E[M]$ is

$$E[M] \quad = qE[N_{K+1}] + (1-q)E[N_{N-K}]. \tag{2}$$

We model the MS cell movement as the *Gambler's Ruin Problem* [10] to solve $E[N_j]$.

Let α_j be the probability that starting from cell j, the MS will reach state $N+1$ before reaching state 0 (i.e., the MS moves out from the right-hand side). From Figure 4, we obtain the following recurrence relation for α_j

$$\alpha_j = p\alpha_{j+1} + (1-p)\alpha_{j-1} \quad \text{for } j = 1, 2, \cdots, N. \tag{3}$$

Since $\alpha_0 = 0$ and $\alpha_{N+1} = 1$, (3) is solved to yield

$$\alpha_j = \begin{cases} \dfrac{1-\left(\dfrac{1-p}{p}\right)^j}{1-\left(\dfrac{1-p}{p}\right)^{N+1}} & \text{for } p \neq \frac{1}{2} \\ \dfrac{j}{N+1} & \text{for } p = \frac{1}{2} \end{cases}. \tag{4}$$

Define a random variable X_y as follows:

$$X_y = \begin{cases} -1 & \text{if the MS moves left at the } y\text{th cell movement} \\ 1 & \text{if the MS moves right at the } y\text{th cell movement} \end{cases}.$$

Then

$$N_j = \min\left\{ n : \sum_{y=1}^{n} X_y = -j \quad \text{or} \quad \sum_{y=1}^{n} X_y = N+1-j \right\}.$$

Note that $E[X_y] = 1 \times p + (-1)(1-p) = 2p-1$ and N_j is a stopping time for X_y's. The $E[X_y]$ value can be a positive or a negative number, and the sign of $E[X_y]$ indicates the direction of MS movement. By using the *Wald's Equation* [10], we have

$$E\left[\sum_{y=1}^{N_j} X_y\right] = (2p-1)E[N_j]. \tag{5}$$

Consider the left-hand side of (5). We have

$$\sum_{y=1}^{N_j} X_y = \begin{cases} N+1-j & \text{with probability } \alpha_j \\ -j & \text{with probability } 1-\alpha_j \end{cases},$$

or

$$E\left[\sum_{y=1}^{N_j} X_y\right] = (N+1-j)\alpha_j + (-j)(1-\alpha_j) = (N+1)\alpha_j - j. \tag{6}$$

Substituting (6) into (5) to yield

$$E[N_j] = \frac{(N+1)\alpha_j - j}{2p-1}. \tag{7}$$

If p approaches $\frac{1}{2}$, $E[N_j]$ can be derived by applying the *L'Hospital's Rule* [11] to (7), which yields

$$\lim_{p\to\frac{1}{2}} E[N_j] = (N+1)j - j^2. \tag{8}$$

To derive q, Figure 5 modifies the state diagram in Figure 4 by removing the absorbing states 0 and $N+1$, and adding the transitions from state N to state $K+1$ (with probability p) and from state 1 to state $N-K$ (with probability $1-p$). When the MS moves out of the current LA from the right-hand side (the left-hand side), the process moves from state N to state $K+1$ (from state 1 to state $N-K$). In other words, the MS moves from cell N (cell 1) of the old LA to cell $K+1$ (cell $N-K$) of the new LA. In this case, the MS would leave

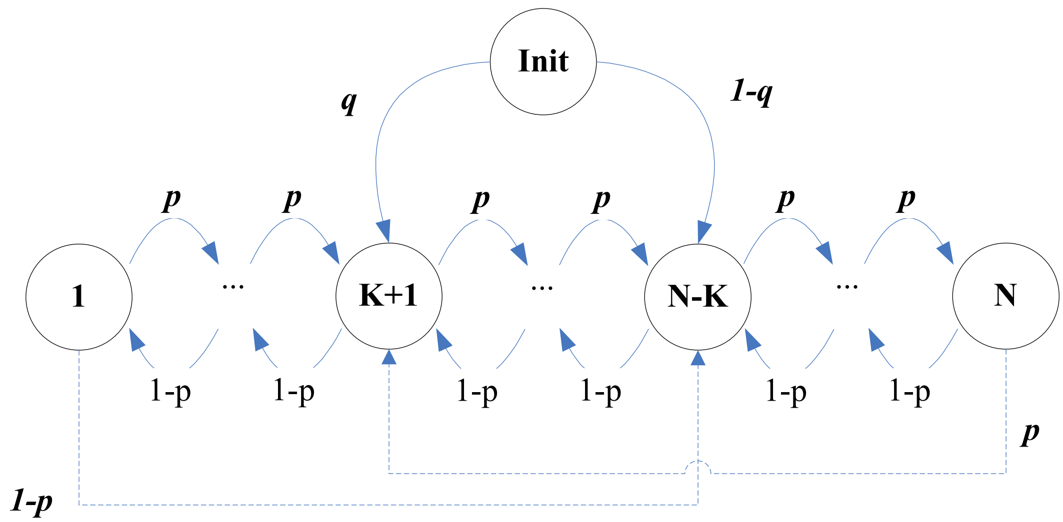

Figure 5: Modified State Transition Diagram for K-degree Overlapping LA Configuration (the LA Size is N, and $0 \leq K < \frac{N}{2}$)

the current LA from the right-hand side boundary and move to cell $K+1$ of new LA with probability $q\alpha_{K+1}$ (i.e., the probability that the MS moves into entrance cell $K+1$ and then moves out the LA from the right-hand side) plus $(1-q)\alpha_{N-K}$ (i.e., the probability that the MS moves into entrance cell $N-K$ and then moves out the LA from the right-hand side). Since the MS moves from state $Init$ to state $K+1$ with probability q, we have the following equation

$$q = q\alpha_{K+1} + (1-q)\alpha_{N-K},$$

or equivalently

$$q = \frac{\alpha_{N-K}}{1-\alpha_{K+1}+\alpha_{N-K}}. \tag{9}$$

Substituting (4) and (7) - (9) into (2), $E[M]$ is expressed as

$$E[M] = \begin{cases} \left(\frac{\alpha_{N-K}}{1-\alpha_{K+1}+\alpha_{N-K}}\right)\left[\frac{(N+1)\alpha_{K+1}-K-1}{2p-1}\right] \\ \quad + \left(\frac{1-\alpha_{K+1}}{1-\alpha_{K+1}+\alpha_{N-K}}\right)\left[\frac{(N+1)\alpha_{N-K}-N+K}{2p-1}\right] & \text{for } p \neq \frac{1}{2} \\ (N-K)\times(K+1) & \text{for } p = \frac{1}{2} \end{cases}, \tag{10}$$

where

$$\alpha_j = \begin{cases} \dfrac{1-\left(\dfrac{1-p}{p}\right)^j}{1-\left(\dfrac{1-p}{p}\right)^{N+1}} & \text{for } p \neq \frac{1}{2} \\ \dfrac{j}{N+1} & \text{for } p = \frac{1}{2} \end{cases} .$$

2.2 Case 2: $\frac{N}{2} \leq K < N$

In this case, an entrance cell is covered by two or more LAs. Therefore, after the MS leaves the old LA, an LA selection policy is required to select the new LA. The $E[M]$ values for the four policies are derived as follows.

- MaxOL: The MS always chooses a new LA with maximum overlapping with the old LA. When the MS moves out of LA i from the right-hand side (the left-hand side), it registers to LA $i+1$ (LA $i-1$). Clearly, the LA selected in this policy is the same as that selected in case $0 \leq K < \frac{N}{2}$ described in Section 2.1. Therefore, the expected number of MS cell movement in an LA is expressed in (10).

- Central: After moving out of LA i, the MS always registers to the LA whose central cell is closest to the entrance cell. The selected LA is the $\left\lceil \frac{N_O}{2} \right\rceil$-th LA away from LA i. That is, the entrance cell is cell $N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)$ of LA $i+\left\lceil \frac{N_O}{2} \right\rceil$ in the right-hand side or cell $\left\lceil \frac{N_O}{2} \right\rceil (N-K)$ of LA $i-\left\lceil \frac{N_O}{2} \right\rceil$ in the left-hand side. The state transition diagram for the Central policy is shown in Figure 6, where the MS moves from state *Init* to state $N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)$ with probability q' and to state $\left\lceil \frac{N_O}{2} \right\rceil (N-K)$ with probability $1-q'$. Following the derivation in Section 2.1, we have

$$q' = \frac{\alpha_{\left\lceil \frac{N_O}{2} \right\rceil (N-K)}}{1-\alpha_{N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)} + \alpha_{\left\lceil \frac{N_O}{2} \right\rceil (N-K)}}$$

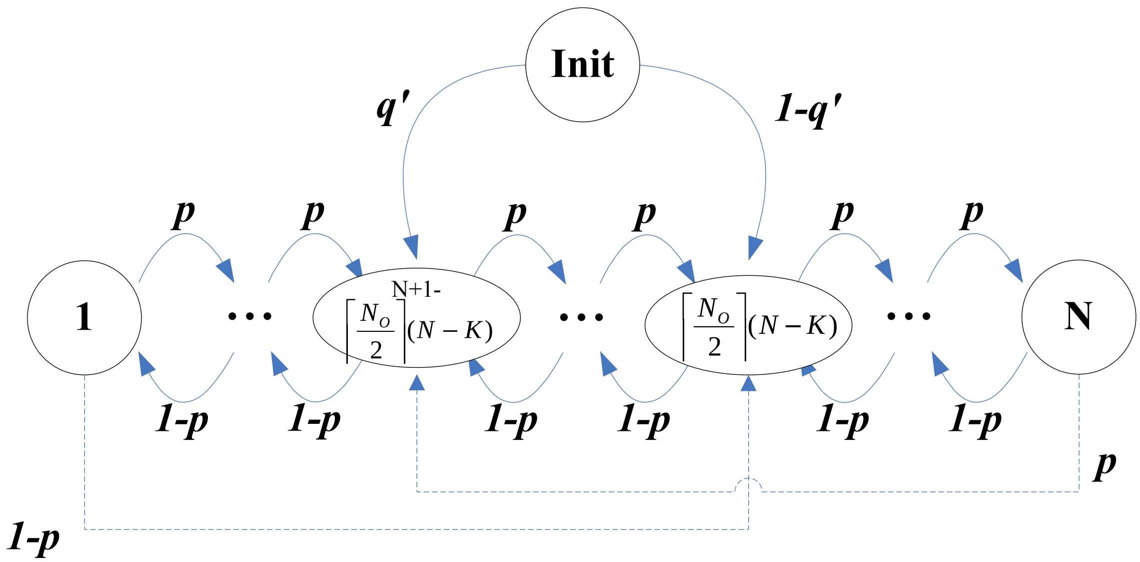

Figure 6: State Transition Diagram for the Central Policy ($\frac{N}{2} \leq K < N$)

and the expected number

$$
\begin{aligned}
& E[M] \\
& = q'E\left[N_{N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)}\right] + (1-q')E\left[N_{\left\lceil \frac{N_O}{2} \right\rceil (N-K)}\right] \\
& = \begin{cases}
\left[\dfrac{\alpha_{\left\lceil \frac{N_O}{2} \right\rceil (N-K)}}{1-\alpha_{N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)} + \alpha_{\left\lceil \frac{N_O}{2} \right\rceil (N-K)}}\right] \\
\quad \times \left\{\dfrac{(N+1)\left(\alpha_{N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)} - 1\right) + \left\lceil \frac{N_O}{2} \right\rceil (N-K)}{2p-1}\right\} \\
+ \left[\dfrac{1-\alpha_{N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)}}{1-\alpha_{N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)} + \alpha_{\left\lceil \frac{N_O}{2} \right\rceil (N-K)}}\right] \\
\quad \times \left[\dfrac{(N+1)\alpha_{\left\lceil \frac{N_O}{2} \right\rceil (N-K)} - \left\lceil \frac{N_O}{2} \right\rceil (N-K)}{2p-1}\right] & \text{for } p \neq \frac{1}{2} \\
\left[\left\lceil \frac{N_O}{2} \right\rceil (N-K)\right] \times \left[N+1-\left\lceil \frac{N_O}{2} \right\rceil (N-K)\right] & \text{for } p = \frac{1}{2}
\end{cases}
\end{aligned} \quad (11)
$$

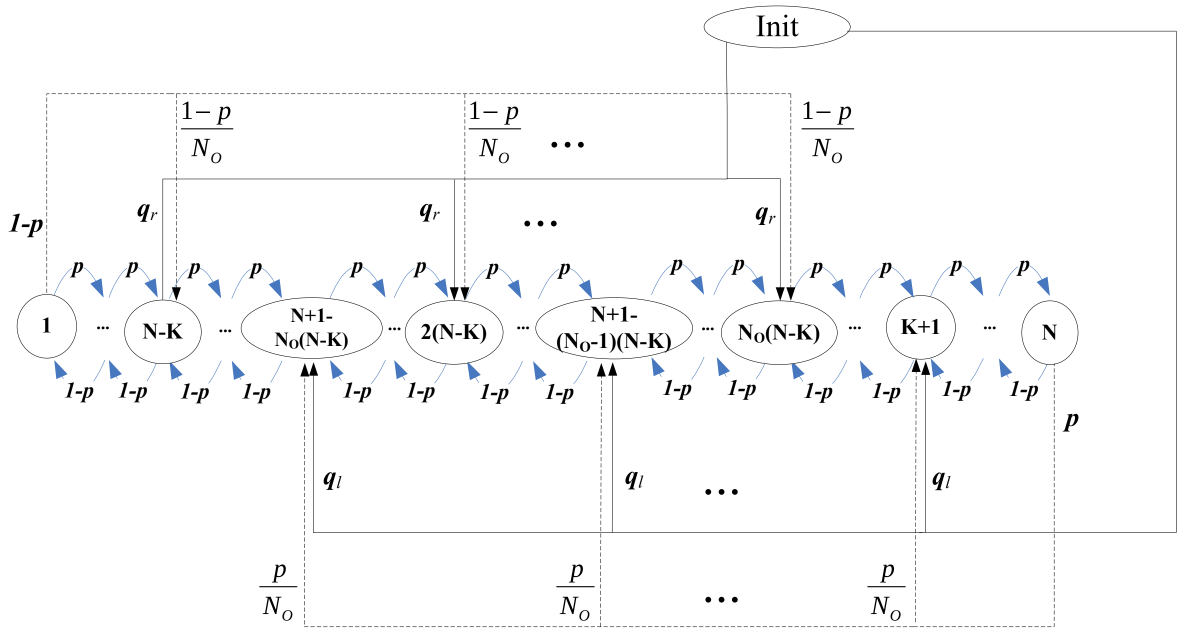

Figure 7: State Transition Diagram for the Random Policy ($\frac{N}{2} \le K < N$)

- Random: The MS randomly registers to one LA covering the entrance cell. Let q_l (q_r) be the probability that the MS moves from a left-hand side LA (right-hand side LA) to the new LA through the entrance cell. The state transition diagram for the Random policy is shown in Figure 7, where the MS moves from state $Init$ to state $N+1-m(N-K)$ with probability q_l and to state $m(N-K)$ with probability q_r, where $1 \le m \le N_O$. From Figure 7, $E[M]$ is expressed as

$$E[M] \quad = q_l \left\{ \sum_{m=1}^{N_O} E\left[N_{N+1-m(N-K)}\right] \right\} + q_r \left\{ \sum_{m=1}^{N_O} E\left[N_{m(N-K)}\right] \right\} . \tag{12}$$

Since the entrance cell is covered by N_O LAs, when the MS leaves the old LA, the process moves from state N (state 1) to state $N+1-m(N-K)$ (state $m(N-K)$) with probability $\frac{p}{N_O}$ ($\frac{1-p}{N_O}$), where $1 \le m \le N_O$; see Figure 7. The balance equations for probabilities q_l and q_r are

$$\begin{aligned}
q_l &= \left(\frac{1}{N_O}\right)\left[q_l \sum_{m=1}^{N_O} \alpha_{N+1-m(N-K)} + q_r \sum_{m=1}^{N_O} \alpha_{m(N-K)}\right] \\
q_r &= \left(\frac{1}{N_O}\right)\left\{\ q_l \sum_{m=1}^{N_O}\left[1-\alpha_{N+1-m(N-K)}\right] + q_r \sum_{m=1}^{N_O}\left[1-\alpha_{m(N-K)}\right]\ \right\} \\
1 &= N_O\left(q_l + q_r\right)
\end{aligned}$$

or equivalently

$$\left.\begin{aligned}
q_l &= \frac{\sum_{m=1}^{N_O} \alpha_{m(N-K)}}{N_O\left\{N_O + \sum_{m=1}^{N_O}\left[\alpha_{m(N-K)} - \alpha_{N+1-m(N-K)}\right]\right\}} \\
q_r &= \frac{1}{N_O} - q_l
\end{aligned}\right\}. \tag{13}$$

Substituting (13) into (12), $E[M]$ is expressed as

$$E[M] = \begin{cases}
\sum_{m=1}^{N_O}\left\{\begin{array}{l} q_l\left(\frac{(N+1)\alpha_{N+1-m(N-K)} - [N+1-m(N-K)]}{2p-1}\right) \\ \quad + \left(\frac{1}{N_O} - q_l\right)\left[\frac{(N+1)\alpha_{m(N-K)} - m(N-K)}{2p-1}\right] \end{array}\right\} & \text{for } p \neq \frac{1}{2} \\
\sum_{m=1}^{N_O} \frac{[m(N-K)] \times [N+1-m(N-K)]}{N_O} & \text{for } p = \frac{1}{2}
\end{cases}, \tag{14}$$

where

$$q_l = \frac{\sum_{m=1}^{N_O} \alpha_{m(N-K)}}{N_O\left\{N_O + \sum_{m=1}^{N_O}\left[\alpha_{m(N-K)} - \alpha_{N+1-m(N-K)}\right]\right\}}.$$

- MinOL: After leaving LA i, the MS chooses the farthest LA of the entrance cell from LA i. The entrance cell is cell $N+1-N_O(N-K)$ of LA $i+N_O$ in the right-hand side or cell $N_O(N-K)$ of LA $i-N_O$ in the left-hand side. The state transition diagram for the MinOL policy is shown in Figure 8. In this figure,

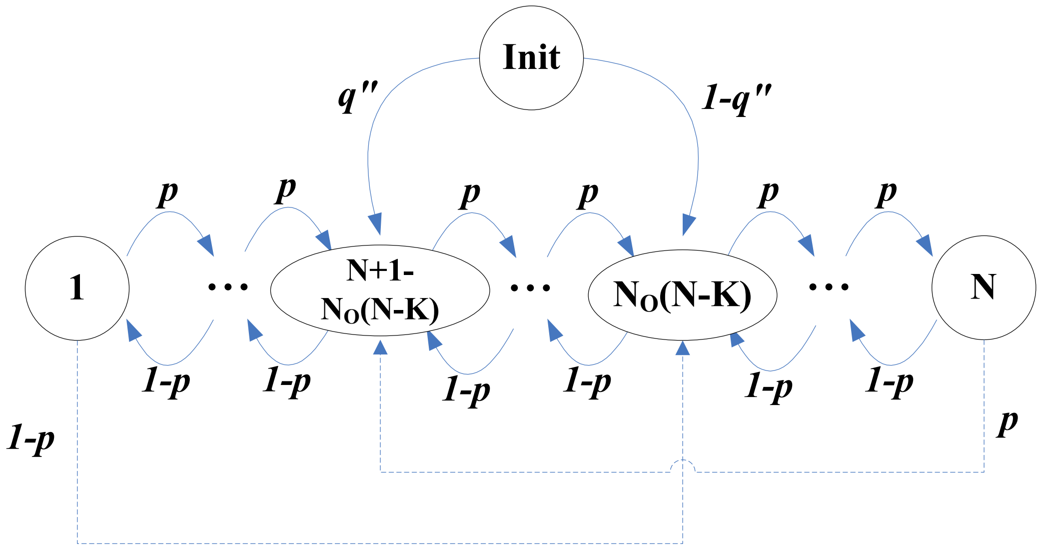

Figure 8: State Transition Diagram for the MinOL Policy ($\frac{N}{2} \leq K < N$)

$$q'' = \frac{\alpha_{N_O(N-K)}}{1 - \alpha_{N+1-N_O(N-K)} + \alpha_{N_O(N-K)}}, \tag{15}$$

and $E[M]$ is expressed as

$$\begin{aligned} E[M] &= q''E\left[N_{N+1-N_O(N-K)}\right] + (1-q'')E\left[N_{N_O(N-K)}\right] \\ &= \begin{cases} \left(\dfrac{\alpha_{N_O(N-K)}}{1 - \alpha_{N+1-N_O(N-K)} + \alpha_{N_O(N-K)}}\right) \\ \quad \times \left\{\dfrac{(N+1)\alpha_{N+1-N_O(N-K)} - [N+1-N_O(N-K)]}{2p-1}\right\} \\ \quad + \left(\dfrac{1 - \alpha_{N+1-N_O(N-K)}}{1 - \alpha_{N+1-N_O(N-K)} + \alpha_{N_O(N-K)}}\right) \\ \quad \times \left[\dfrac{(N+1)\alpha_{N_O(N-K)} - N_O(N-K)}{2p-1}\right] & \text{for } p \neq \frac{1}{2} \\ [N_O(N-K)] \times [N+1-N_O(N-K)] & \text{for } p = \frac{1}{2} \end{cases} . \end{aligned} \tag{16}$$

Note that our analytic results have been validated against the simulation experiments (see Table 1). The errors between the analytic and simulation models are under 1%.

3 Performance Evaluation

This section studies the performance of the LA selection policies (i.e., MaxOL, Central, Random and MinOL). Specifically, we investigate the expected number $E[M]$ of cell movement

Table 1: Analytic and Simulation Results ($N = 15$)

K	0	4	8	12
$p = 0.5$				
E[M] (Analytic)	15	55	63	39
E[M] (Simulation)	15.022	54.979	62.913	38.845
Error	-0.152%	0.037%	0.136%	0.396%
$p = 0.7$				
E[M] (Analytic)	15	26.716	17.482	7.499
E[M] (Simulation)	15.032	26.622	17.458	7.501
Error	-0.214%	0.351%	0.141%	-0.023%

(a) The MaxOL Policy

K	0	4	8	12
$p = 0.5$				
E[M] (Analytic)	15	55	63	63
E[M] (Simulation)	14.987	55.060	62.925	62.927
Error	0.084%	-0.109%	0.118%	0.115%
$p = 0.7$				
E[M] (Analytic)	15	26.716	17.482	22.380
E[M] (Simulation)	15.032	26.622	17.465	22.330
Error	-0.214%	0.351%	0.100%	0.223%

(b) The Central Policy

K	0	4	8	12
$p = 0.5$				
E[M] (Analytic)	15	55	45.5	45
E[M] (Simulation)	15.003	54.899	45.129	44.833
Error	-0.020%	0.181%	0.814%	0.370%
$p = 0.7$				
E[M] (Analytic)	15	26.716	21.818	18.608
E[M] (Simulation)	15.025	26.738	21.992	18.579
Error	-0.169%	-0.083%	-0.798%	0.157%

(c) The Random Policy

K	0	4	8	12
$p = 0.5$				
E[M] (Analytic)	15	55	28	15
E[M] (Simulation)	15.003	54.899	27.969	14.995
Error	-0.020%	0.181%	0.107%	0.031%
$p = 0.7$				
E[M] (Analytic)	15	26.716	24.137	15
E[M] (Simulation)	15.025	26.738	24.136	15.008
Error	-0.169%	-0.083%	0.007%	-0.056%

(d) The MinOL Policy

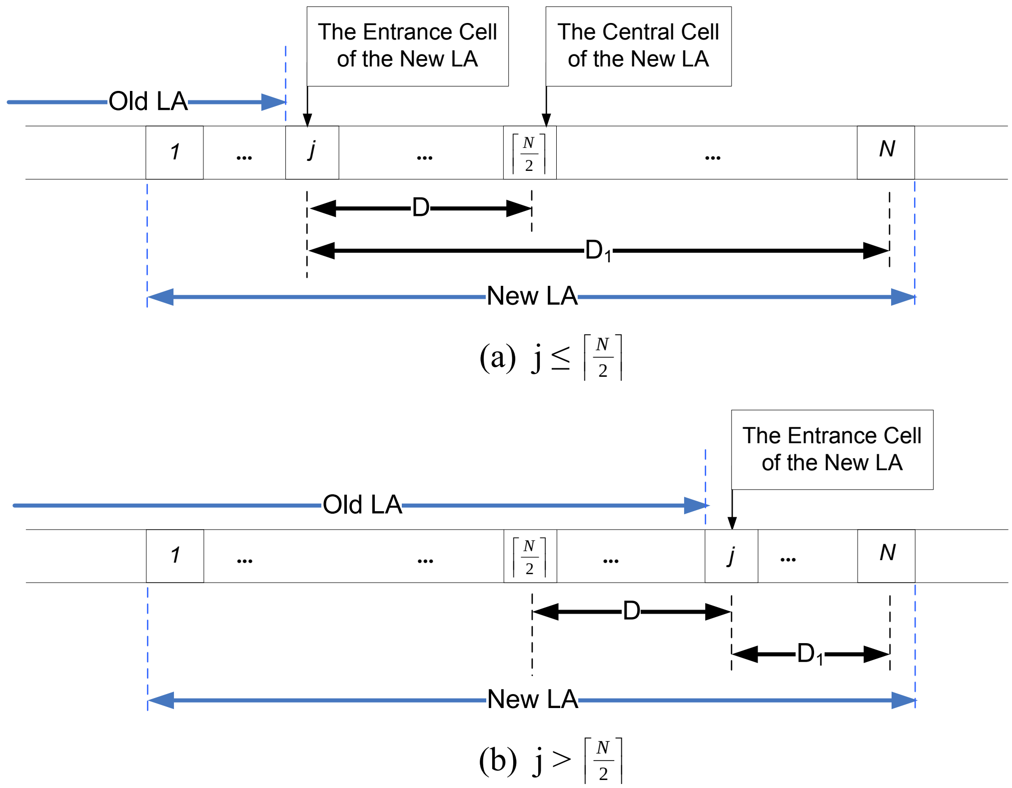

Figure 9: Distances D and D_1 in the New LA

in an LA before an MS leaves the LA. It is clear that the larger the $E[M]$ value, the better the performance. In our numerical examples, the LA size $N = 15$. The results for other N values are similar and are omitted.

We first note that due to the symmetric cell structure in each LA, the effect for p is identical to that for $1 - p$. Therefore, it suffices to consider $0.5 \leq p \leq 1$. Denote D as the distance (the number of cells) between the entrance cell (cell j in Figure 9) and the central cell of the new LA (cell $\left\lceil \frac{N}{2} \right\rceil$ in Figure 9). Denote D_1 as the distance between the entrance cell and the new LA's right-hand side (left-hand side) boundary cell if the MS enters the new LA from the left-hand side (right-hand side) of the LA (cell N in Figure 9). Note that $E[M]$ is affected by the ping-pong effect and moving-to-one-direction effect. Besides p, these effects are determined by D and D_1. The smaller the D value, the less significant the ping-pong

Table 2: The Distance D between the Entrance Cell and the Cental Cell of the New LA ($N = 15$)

K	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
MaxOL	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7
Cental	7	6	5	4	3	2	1	0	1	2	2	0	1	0	0
Random	7	6	5	4	3	2	1	0	3.5	3	4	2.7	3.8	3.4	3.7
MinOL	7	6	5	4	3	2	1	0	6	4	7	4	7	6	7

Table 3: The Distance D_1 ($N = 15$)

K	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
MaxOL	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
Cental	14	13	12	11	10	9	8	7	6	5	9	7	8	7	7
Random	14	13	12	11	10	9	8	7	9.5	8	9	7	8	7	7
MinOL	14	13	12	11	10	9	8	7	13	11	14	11	14	13	14

effect. The larger the the D_1 value, the less significant the moving-to-one-direction effect. Tables 2 and 3 list D and D_1 as functions of K. These D and D_1 values are computed from the analytic model, and are validated by the simulation experiments.

Figures 10 - 13 plot $E[M]$ as a function of K and p. Consider $0.5 \leq p \leq 1$. When p is small, the ping-pong effect is significant, and $E[M]$ is likely to increase as D decreases. When $p = 0.5$, $E[M]$ is a decreasing function of D (see Figure 10 and Table 2). When $p = 1$, the MS always moves to one direction. Therefore, $E[M]$ increases as D_1 increases (see Figure 13 and Table 3). When p increases, the ping-pong effect (impact of D) becomes insignificant, and the moving-to-one-direction effect (impact of D_1) becomes significant. Consider the MaxOL policy, $E[M]$ increases and then decreases as K increases. For $p = 0.5, 0.7, 0.9, 1$, the maximal $E[M]$ values occur when $K = 7, 3, 1, 0$, respectively. For a maximal $E[M]$ value, the corresponding K value is called the *best overlapping degree*. As p increases, the impact of D_1 becomes more significant, and the best overlapping degree decreases. Figures 14 - 17 plot $E[M]$ as a function of K and p with $N = 100$. The results

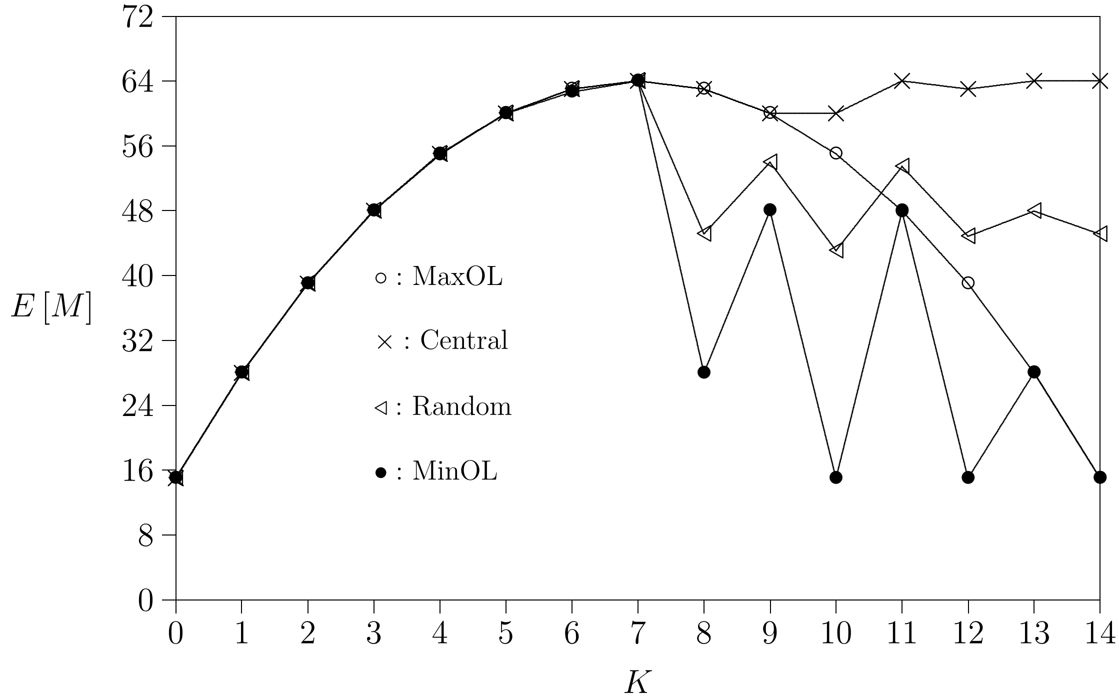

Figure 10: Effects of K on $E\,[M]$ for $p = 0.5$

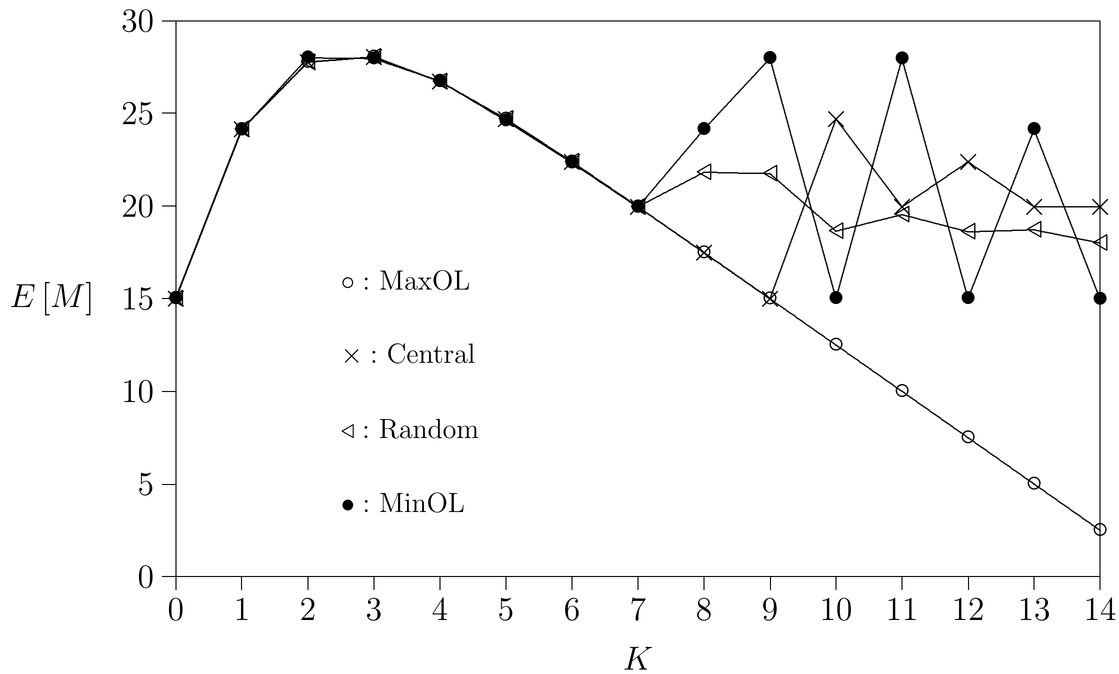

Figure 11: Effects of K on $E\,[M]$ for $p = 0.7$

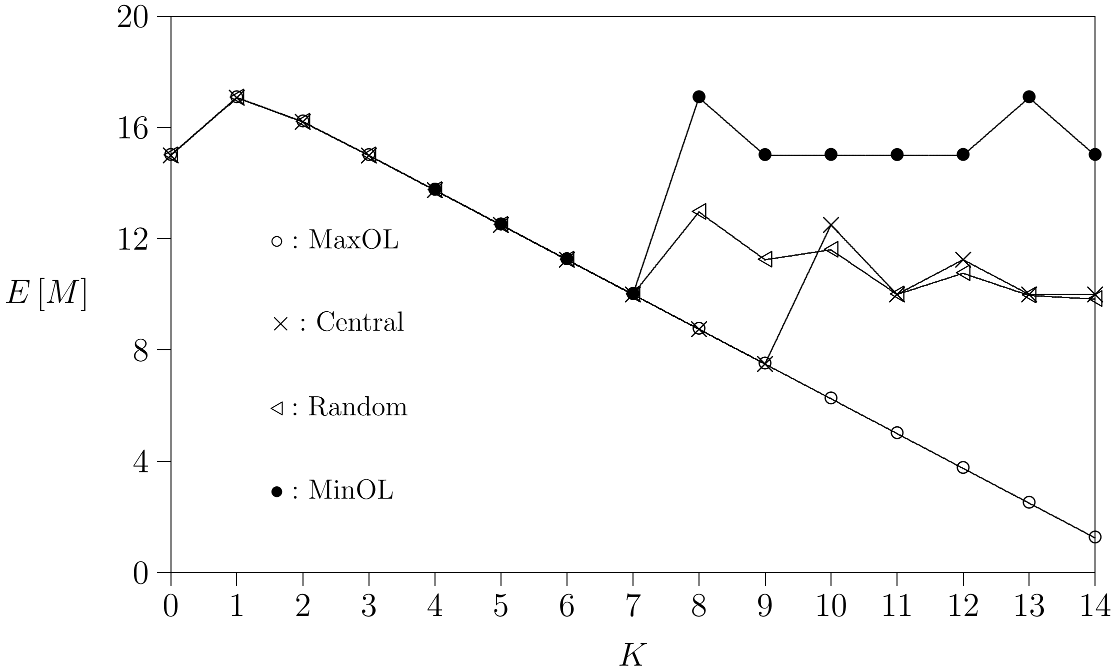

Figure 12: Effects of K on $E\,[M]$ for $p = 0.9$

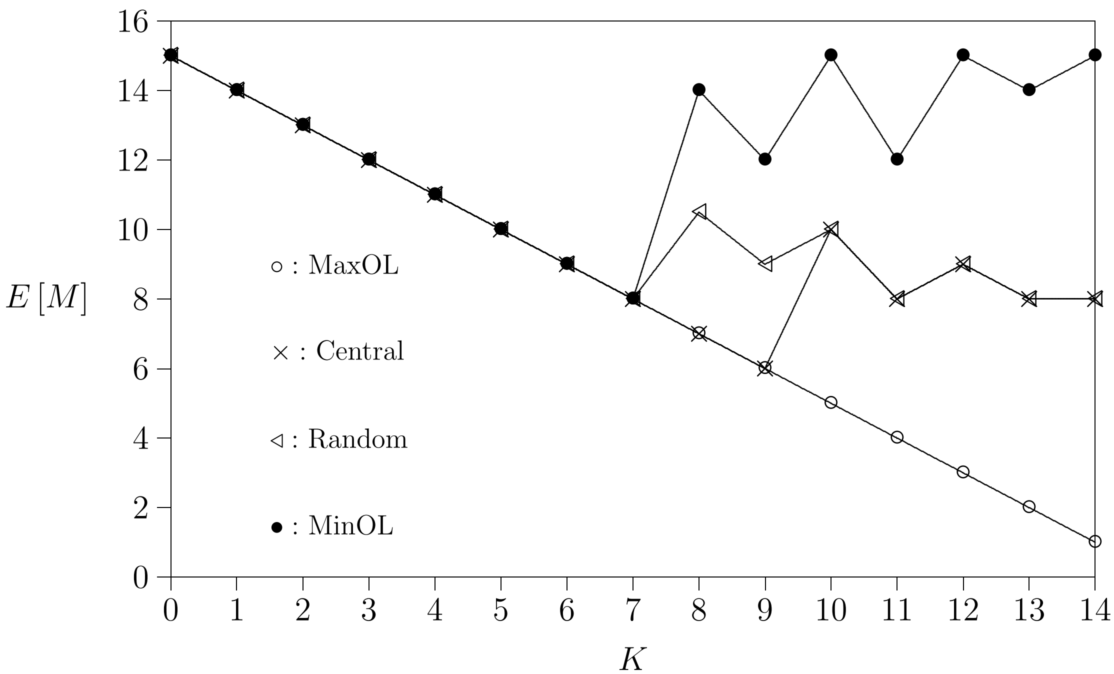

Figure 13: Effects of K on $E\,[M]$ for $p = 1.0$

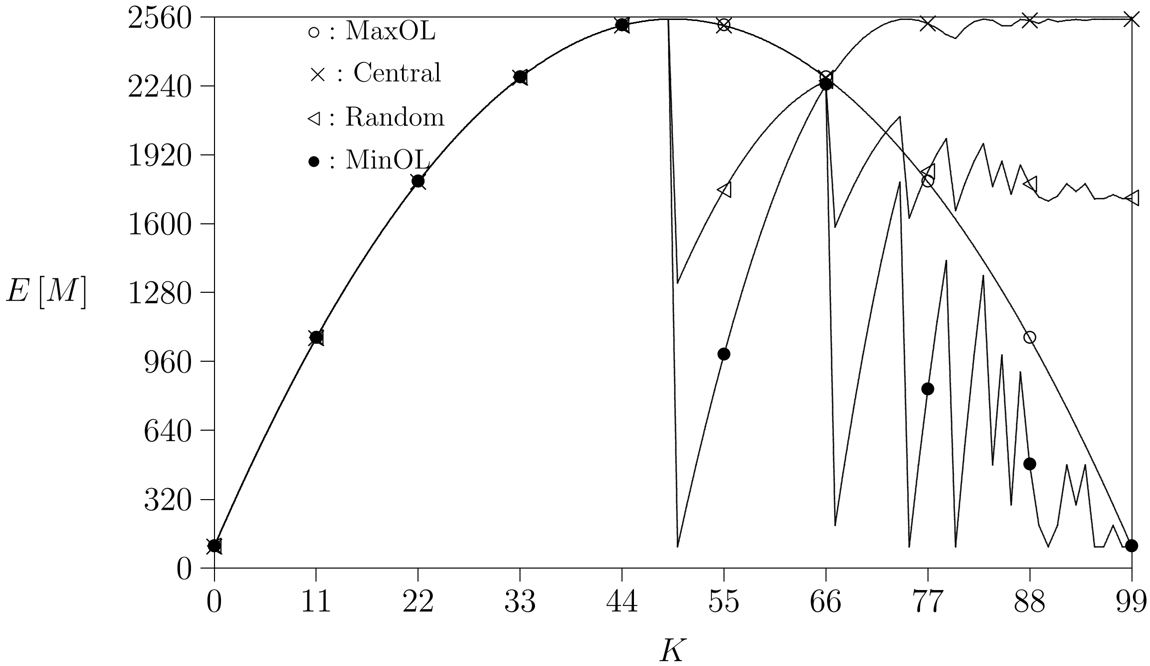

Figure 14: Effects of K on $E[M]$ for $p = 0.5$ and $N = 100$

are similar to that for $N = 15$, and the details are omitted.

From Figures 10 - 17, we observe that when $0 \leq K < N/2$, all four policies result in same performance. When $N/2 \leq K < N$, the *Central* policy makes sense for $p = 0.5$, while the *MinOL* policy is more appropriate for $p = 1.0$. For all policies, these figures show an apparent result that for the same K value, $E[M]$ decreases as p increases. By differentiating the analytic equations (equations (10), (11), (14) and (16)), we can formally prove that the best overlapping degree always occurs when $0 \leq K < \frac{N}{2}$. That is, in one-dimensional overlapping LA configuration, it suffices to consider the configurations for $K < \frac{N}{2}$.

4 Conclusions

This paper studied four LA selection policies for overlapping LA configuration: *Maximum Overlapping* (MaxOL), *Central*, *Random* and *Minimum Overlapping* (MinOL) policies. We proposed an analytic model to investigate the $E[M]$ performance of these policies for the one-dimensional overlapping LA configuration. The analytic results were validated against

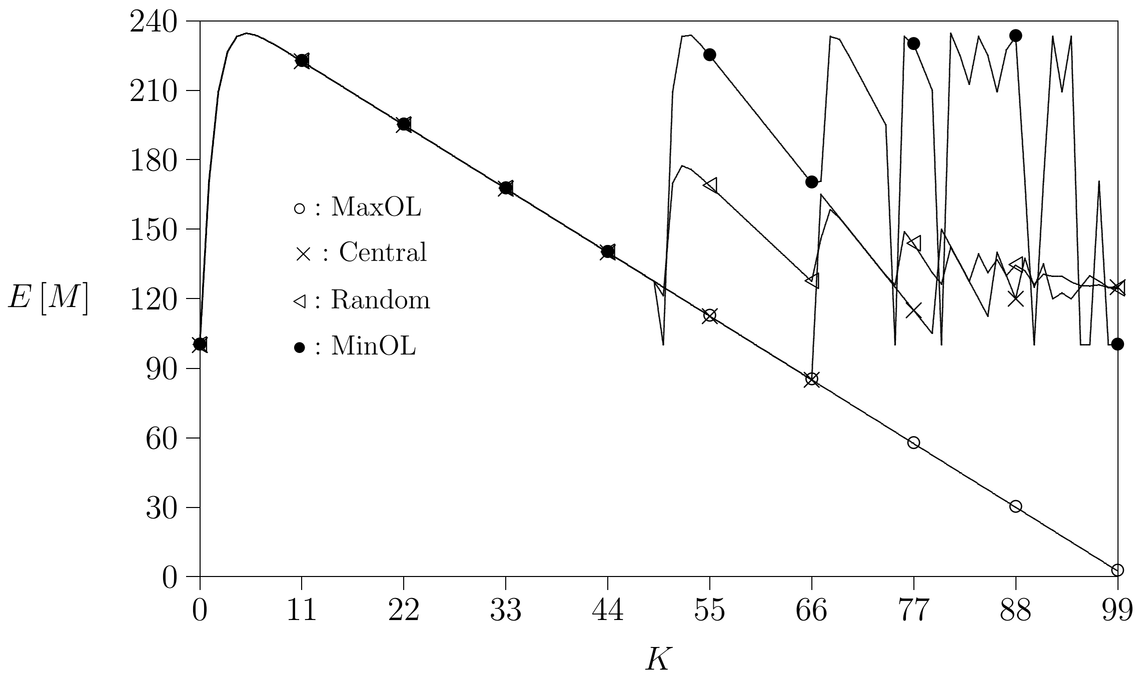

Figure 15: Effects of K on $E[M]$ for $p = 0.7$ and $N = 100$

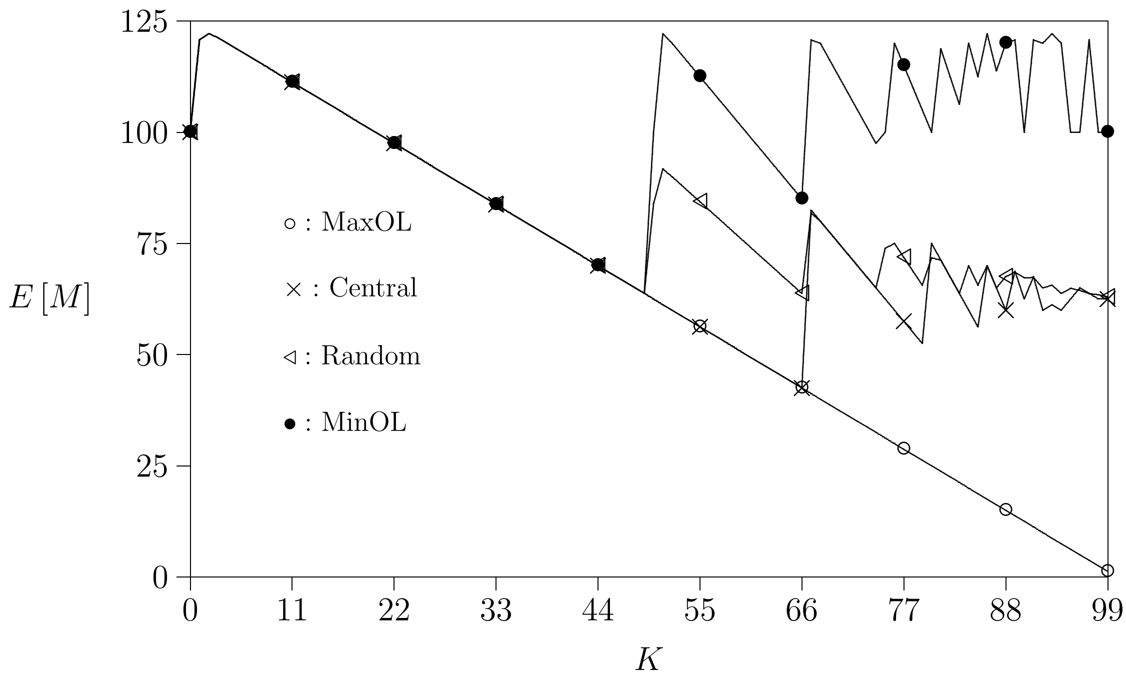

Figure 16: Effects of K on $E[M]$ for $p = 0.9$ and $N = 100$

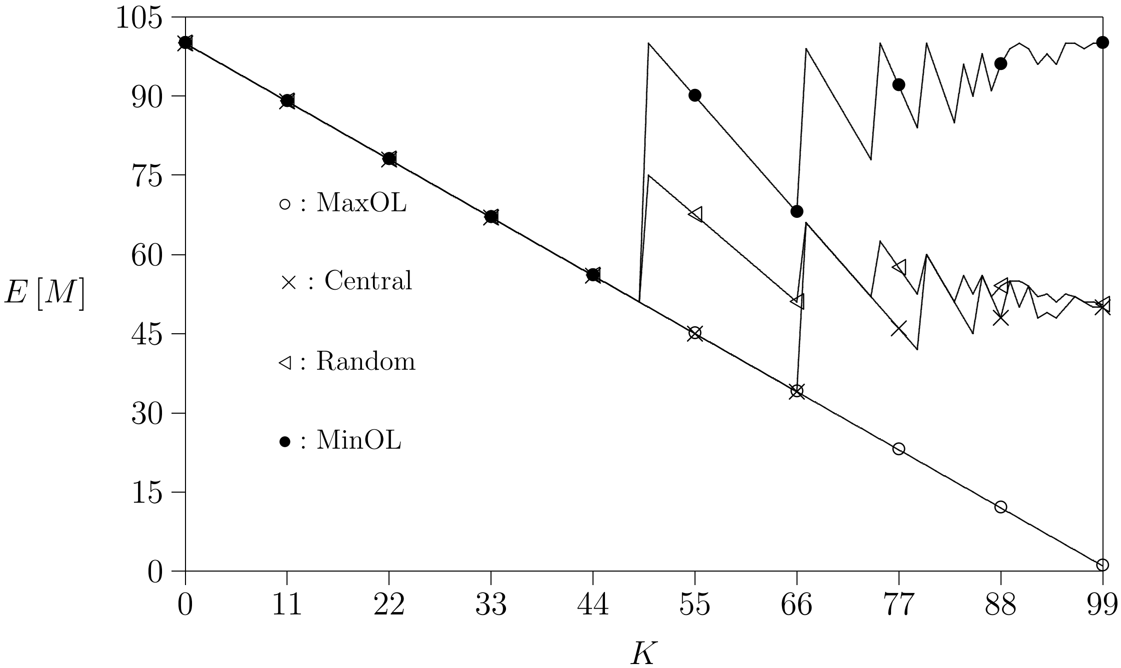

Figure 17: Effects of K on $E[M]$ for $p = 1.0$ and $N = 100$

the simulation experiments.

Our study for the one-dimensional overlapping LA configuration is more general than the previous studies. For specific scenarios in the previous studies, their results are consistent with ours (which validates that our results are correct). The major difference between our model and those in the previous studies is the setup for the routing probability p. All previous studies assumed that at each movement, the MS moves to any of its neighboring cells with the same routing probability. In our study, we assume that the MS moves to each of its neighboring cells with different probabilities (i.e., $0 < p \leq 1.0$). Furthermore, the overlapping degree K in our study can be arbitrary (i.e., $0 \leq K < N$). The previous studies made restrictive assumption where $0 \leq K < \frac{N}{2}$. We note that there is a typo in equation (11) of [5]. Specifically, the $(2d - w + 1)$ term in the right-hand side of the equation should be rewritten as $(2d - w - 1)$.

Our study indicates the following results.

- It suffices to consider the configurations for $0 \leq K < \frac{N}{2}$, and all four policies result in same performance in these configurations. When $N/2 \leq K < N$, the *Central* policy makes sense for $p = 0.5$, while the *MinOL* policy is more appropriate for $p = 1.0$.

- When the routing probability $p \in [0.5, 1]$, the $E[M]$ value decreases as p increases. When $p = 1$ (i.e., the mobile telecommunications network is deployed in the highway), the $E[M]$ performance for $K = 0$ is always better than that for $K > 0$.

From the numerical comparison of the four LA selection policies, our study indicates that in practical scenarios, the best overlapping degree occurs for $0 \leq K < \frac{N}{2}$, and it suffices to consider the MaxOL policy. This important result has not been reported in the literature. In the future, we will extend our one-dimensional model for two-dimensional LA layout.

A Notation

- N: the number of cells in each LA
- K: the number of overlapping cells between two neighboring LAs
- p: the routing probability that the MS moves to the right-hand side neighboring cell
- N_O: the number of the LAs covering the entrance cell
- α_j: the probability that starting from cell j, the MS leaves the old LA from the right-hand side
- $E[N_j]$: the expected number of cell movement, starting from cell j and before the MS leaves the LA
- $E[X_y]$: the mean of the MS's yth cell movement
- q: the probability that the MS enters the new LA from a left-hand side LA for case $0 \leq K < \frac{N}{2}$
- q': the probability that in the Central policy, the MS enters the new LA from a left-hand side LA for case $\frac{N}{2} \leq K < N$

- q_l: the probability that in the Random policy, the MS enters the new LA from a left-hand side LA for case $\frac{N}{2} \leq K < N$

- q_r: the probability that in the Random policy, the MS enters the new LA from a right-hand side LA for case $\frac{N}{2} \leq K < N$

- q'': the probability that in the MinOL policy, the MS enters the new LA from a left-hand side LA for case $\frac{N}{2} \leq K < N$

- $E[M]$: the expected number of cell movement before the MS leaves the LA

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