# CS5319 Advanced Discrete Structure

Homework 3

Due: 1:10 pm, November 8, 2010 (before class)

1. Solve the following recurrence relations.

   (a) $\begin{cases} a_{n+1} - a_n = 3n + 2, & n \geq 0 \\ a_0 = 1 \end{cases}$

   (b) $\begin{cases} a_{n+1} - a_n = 2n^2 - n, & n \geq 0 \\ a_0 = 3 \end{cases}$

2. Solve the following recurrence relations.

   (a) $\begin{cases} a_{n+2} - 2a_{n+1} + a_n = 2^n, & n \geq 0 \\ a_0 = 1 \\ a_1 = 2 \end{cases}$

   (b) $\begin{cases} a_{n+1} = 2a_n - b_n + 2, & n \geq 0 \\ b_{n+1} = -a_n + 2b_n - 1, & n \geq 0 \\ a_0 = 0 \\ b_0 = 1 \end{cases}$

3. Suppose that $A(x)$ is the generating function for the sequence $(a_0, a_1, a_2, \ldots)$.

   (a) Find the generating function for the sequence $(s_0, s_1, s_2, \ldots)$, where

   $$s_n = \sum_{i=0}^{n} a_i.$$

   (b) Find the generating function for the sequence

   $$(0, 1^2, 1^2 + 2^2, 1^2 + 2^2 + 3^2, \ldots).$$

4. (a) Let $b_n$ denote the number of $n$-bit binary strings in which the pattern 01001 *first* occurs at the $n$th bit. Find the generating function for $(b_0, b_1, \ldots)$.

   (b) Let $c_n$ denote the number of $n$-bit binary strings in which the pattern 01001 occurs at the $n$th bit. Using the result of part (a), find the generating function for $(c_0, c_1, \ldots)$.

5. Let $d_n$ be the number of ways to completely cover a $3 \times n$ rectangle with $3 \times 1$ dominoes. Find the generating function for $(d_0, d_1, d_2, \ldots)$.