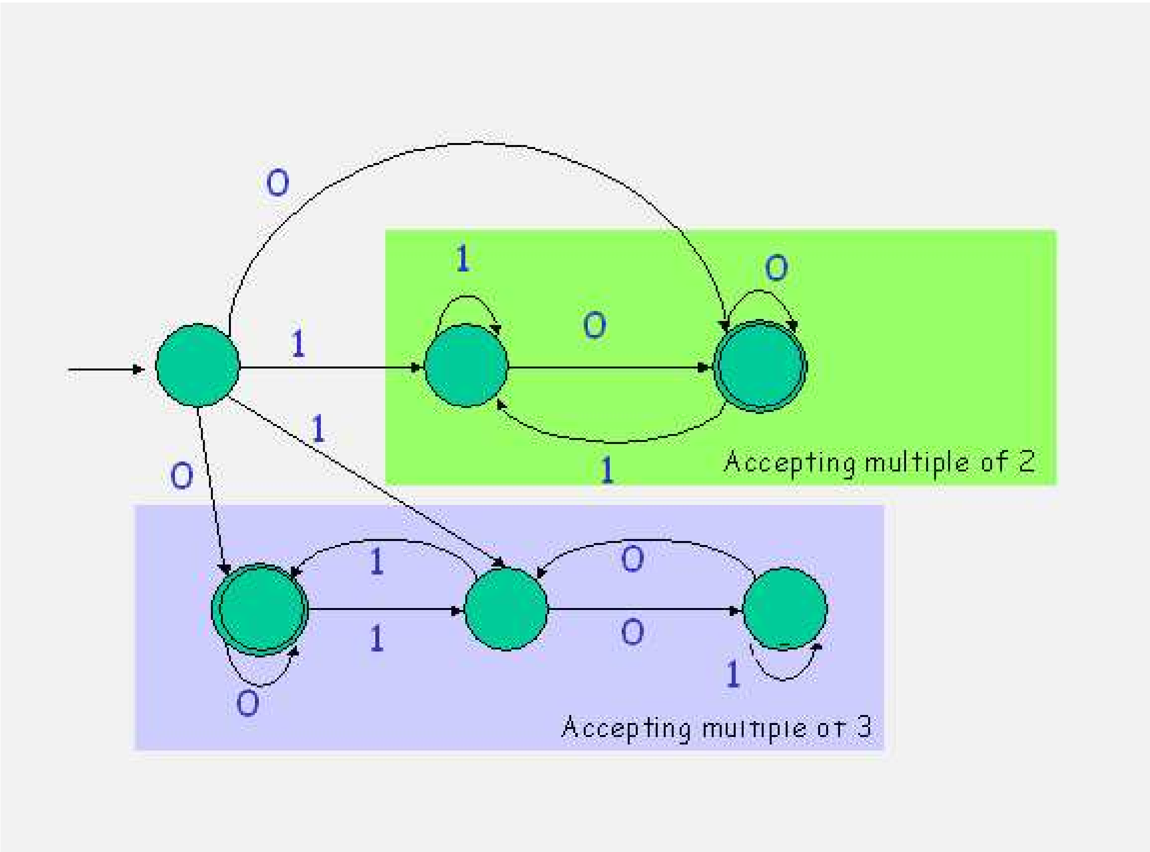
CS5371 Theory of Computation

Mid-term Quiz (Solution)

Part I. Long Questions (Total: 75 marks, 25 marks each)

1. Give the state diagram of a DFA (or an NFA) that accepts only binary strings which represent numbers divisible by two or three. E.g., it accepts `0`, `00`, `10`, `011`, but it rejects the empty string, `1`, `101`, `0111`.

   Answer. We give an NFA that recognizes the above language.

   General comments:

   (a) Most of you can construct the part accepting strings representing a multiple of 2 correctly.

   (b) Some are not familiar how to draw a DFA accepting binary strings which represent a multiple of 3.

   (c) Most common mistake is: accept the empty string ε.

2. State the pumping lemma for regular language. Prove that $\{0^n1^n \mid n >= 0\}$ is not a regular language.

   Answer. The pumping lemma for regular language states that: If L is a regular language, there exists a pumping length p such that if $s \in L$ with $|s| \geq p$, s can be written as xyz satisfying the following three conditions: (`i`) $|xy| \leq p$, (`ii`) $|y| > 0$, and (`iii`) for every $i = 0, 1, 2, ..., xy^iz \in L$.

To show that the language $A = \{0^n 1^n \mid n >= 0\}$ is not regular, suppose on the opposite that it is regular. Then, let p be the pumping length of A, and consider the string $s = 0^p 1^p$ in A. By pumping lemma, s can be written as xyz satisfying the three conditions in the lemma. By our choice of the string, this implies that y must be of the form 0^r for some $1 \leq r \leq p$, since $|xy| \leq p$ (so that xy is a substring of 0^p) and $|y| > 0$. Then, the string $xyyz$ will have more zeros than ones, which implies it is not in A. Thus, a contradiction occurs (as the lemma states that $xyyz$ is in A), so that we conclude A is not regular.

General comments:

(a) Most of you can correctly show the language is not regular.

(b) Some forget that $|y|$ can be of *any* length between 1 to p.

(c) Most common mistake is: forget to state the pumping lemma *explicitly*. Some put the pumping lemma (or part of it) inside the proof, such as starting with "Let $L = \{0^n 1^n \mid n >= 0\}$ be a regular language. Then, there exists a pumping length p ...", so that it is not considered as stating.

3. Prove that $\{0^n 1^n 2^m \mid n, m >= 0\}$ is a CFL.

   Answer. We show that this language is a CFL by giving a CFG that generates it. Intuitively, A generates a string of the form $0^n 1^n$, and B generates a substring of the form 2^n, for all $n \geq 0$.

$$\begin{aligned} &S \to AB \\ &A \to 0A1 \mid \varepsilon \\ &B \to 2B \mid \varepsilon \end{aligned}$$

   General comments:

   (a) Most answer this question correctly.

   (b) Some construct PDA to show that the language is a CFL.

   (c) Most common mistake is: forget to generate ε.

Part II. Short Questions (Total: 25 marks, 5 marks each)

1. Can an ambiguous CFG be converted into Chomsky Normal Form? Explain briefly your answer.

   Answer. Yes. Any CFG can be converted into CNF, no matter it is ambiguous or not.

2. What is the maximum number of language a CFG can generate? Explain briefly your answer.

   Answer. One. A CFG can generate a specific set of strings (which may contain no strings, or infinitely many strings), and this specific set of strings is called the language generated by the CFG.

3. Is it true that if L is a language recognized by an NTM, L can be recognized by a DTM? (No need to explain your answer.)

   Answer. Yes.

4. What is the minimum number of states a DTM can have? Explain briefly your answer.

   Answer. Two. By definition, q_{accept} and q_{reject} are distinct states in the DTM. (Note: q_{start} can be one of these two states.)

5. Let L and L' be two non-regular languages. Is it true that the intersection of L and L' must be non-regular? Explain briefly your answer.

   Answer. No. Let L be the language $\{\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\}$, and let L' be the complement of L. Both languages are non-regular. However, their intersection is $\{\}$ (set with no strings), which is a regular language.