## CS 2336 Discrete Mathematics

Lecture 3 Logic: Rules of Inference

## Outline

- Mathematical Argument
- Rules of Inference

#### Argument

- In mathematics, an argument is a sequence of propositions (called premises) followed by a proposition (called conclusion)
- A valid argument is one that, if all its premises are true, then the conclusion is true
- Ex: "If it rains, I drive to school." "It rains."
  - .: "I drive to school."

## Valid Argument Form

• In the previous example, the argument belongs to the following form:

 $p \rightarrow q$ p $\therefore q$ 

- Indeed, the above form is valid no matter what propositions are substituted to the variables
- This is called a valid argument form

## Valid Argument Form

- By definition, if a valid argument form consists
  - -premises:  $p_1, p_2, \dots, p_k$
  - -conclusion: q

then (  $p_1 \wedge p_2 \wedge ... \wedge p_k$  )  $\rightarrow q~$  is a tautology

- Ex: ( (  $p \rightarrow q$  )  $\land p$  )  $\rightarrow q$  is a tautology
- Some simple valid argument forms, called rules of inference, are derived and can be used to construct complicated argument form

- Modus Ponens (method of affirming)
   premises: p, p → q
   conclusion: q
- 2. Modus Tollens (method of denying)
  premises: ¬q, p→q
  conclusion: ¬p

- 3. Hypothetical Syllogism premises:  $p \rightarrow q, q \rightarrow r$ conclusion:  $p \rightarrow r$
- 4. Disjunctive Syllogism
  premises: ¬p, p∨q
  conclusion: q

5. Addition

premises: p

conclusion:  $p \lor q$ 

6. Simplification
premises: p ∧ q
conclusion: p

7. Conjunction

premises: p, q

conclusion:  $p \land q$ 

8. Resolution premises:  $p \lor q$ ,  $\neg p \lor r$ conclusion:  $q \lor r$ 

## Rules of Inference with Quantifiers

- 9. Universal Instantiation
  premises: ∀x P(x)
  conclusion: P(c), for any c
- 10. Universal Generalization premises: P(c) for any arbitrary c conclusion:  $\forall X P(X)$

## Rules of Inference with Quantifiers

- 11. Existential Instantiation premises:  $\exists x P(x)$ conclusion: P(c), for some element c
- 12. Existential Generalization premises: P(c) for some element c conclusion:  $\exists x P(x)$

# **Applying Rules of Inferences**

- Example 1: It is known that
  - 1. It is not sunny this afternoon, and it is colder than yesterday.
  - 2. We will go swimming only if it is sunny.
  - 3. If we do not go swimming, we will play basketball.
  - 4. If we play basketball, we will go home early.
- Can you conclude "we will go home early"?

- To simplify the discussion, let
  - p := It is sunny this afternoon
  - q := It is colder than yesterday
  - r := We will go swimming
  - s := We will play basketball
  - t := We will go home early
- We will give a valid argument with
   premises: ¬p∧r, r→p, ¬r→s, s→t
   conclusion: t

Step

2. ¬ p

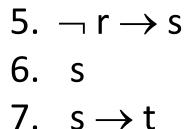
4. ¬ r

3.  $r \rightarrow p$ 

1.  $\neg p \wedge r$ 

#### Reason

- Premise
- Simplification using (1)
- Premise
- Modus Tollens using (2) and (3)
- Premise
  - Modus Ponens using (4) and (5)
- Premise
  - Modus Ponens using (6) and (7)



8. t

# **Applying Rules of Inferences**

- Example 2: It is known that
  - 1. If you send me an email, then I will finish my program.
  - 2. If you do not send me an email, then I will go to sleep early.
  - 3. If I go to sleep early, I will wake up refreshed.
- Can you conclude "If I do not finish my program, then I will wake up refreshed"?

- To simplify the discussion, let
  - p := You send me an email
  - q := I finish my program
  - r := I go to sleep early
  - s := I wake up refreshed
- We will give a valid argument with premises: p→q, ¬p→r, r→s conclusion: ¬q→s

Step	Reason
1. $p \rightarrow q$	Premise
$2. \neg q \rightarrow \neg p$	Contrapositive of (1)
3. $\neg p \rightarrow r$	Premise
4. $\neg q \rightarrow r$	Hypothetical Syllogism by (2) and (3)
5. $r \rightarrow s$	Premise
6. $\neg q \rightarrow s$	Hypothetical Syllogism by (4) and (5)

## **Applying Rules of Inferences**

- Example 3: It is known that
  - 1. A student in this class has not read the book.
  - 2. Everyone in this class passed the first exam.

 Can you conclude that "Someone who passed the first exam has not read the book"?

• To simplify the discussion, let

C(X) := X is a student in the class B(X) := X has read the book P(X) := X passed the first exam

• We will give a valid argument with premises:  $\exists X (C(X) \land \neg B(X)), \forall X (C(X) \rightarrow P(X))$ conclusion:  $\exists X (P(X) \land \neg B(X))$ 

#### Step

1.  $\exists \chi (C(\chi) \land \neg B(\chi))$ 2. C(a)  $\wedge \neg B(a)$ 3. C(a) 4.  $\forall X (C(X) \rightarrow P(X))$ 5. C(a)  $\rightarrow$  P(a) 6. P(a) 7. ¬B(a) 8. P(a)  $\wedge \neg B(a)$ 

Reason Premise **Existential Instantiation** Simplification by (2) Premise Universal Instantiation Modus Ponens by (3) and (5) Simplification by (2) Conjunction by (6) and (7) 9.  $\exists \chi (P(\chi) \land \neg B(\chi))$  Existential Generalization

#### From Sherlock Holmes

• The following is from Silver Blaze, one of Sherlock Holmes stories (written by Sir Arthur Conan Doyle):

Gregory: Is there any other point to which you would wish to draw my attention?

- Holmes: To the curious incident of the dog in the night-time.
- Gregory: The dog did nothing in the night-time.
- Holmes: That was the curious incident.