# CS 2336
# Discrete Mathematics

## Lecture 7

Counting: Basics

# Outline

- Rule of Sum
- Rule of Product
- Principle of Inclusion-Exclusion
- Tree Diagrams

# Rule of Sum

- PizzaHut is currently serving the following kinds of individual meals:

  Pizzas : Supreme, Takoyaki, Kimchi, Hawaiian, Smoked House, Seafood, Veggie Delight, Veggie, Beef

  Pastas : Seafood Marinara, Chicken, Curry Veggie, Seafood Carbonara, Lasagna, Bolognaise

# Rule of Sum

- Bruce wants to try a different meal each day.

  How long does it take for him to try each meal once ?

- There are 9 kinds of pizzas, and 6 kinds of pastas.

  Altogether, there are 9 + 6 = 15 kinds of meal

  ➔ Bruce needs 15 days

# Rule of Sum

- In general, we have the following rule :

**Rule of Sum :**

If one event can occur in m ways and another event can occur in n ways,

then there are m + n ways that one of these events can occur

# Rule of Product

- PizzaHut also offers various side drinks :

  Soup, Coke, Sprite, Coke Zero, Sokenbicha



- Suppose that Bruce wants to include a drink with his individual meal each day (15 of them).

  How many different combinations are there ?

# Rule of Product

- In general, we have the following rule :

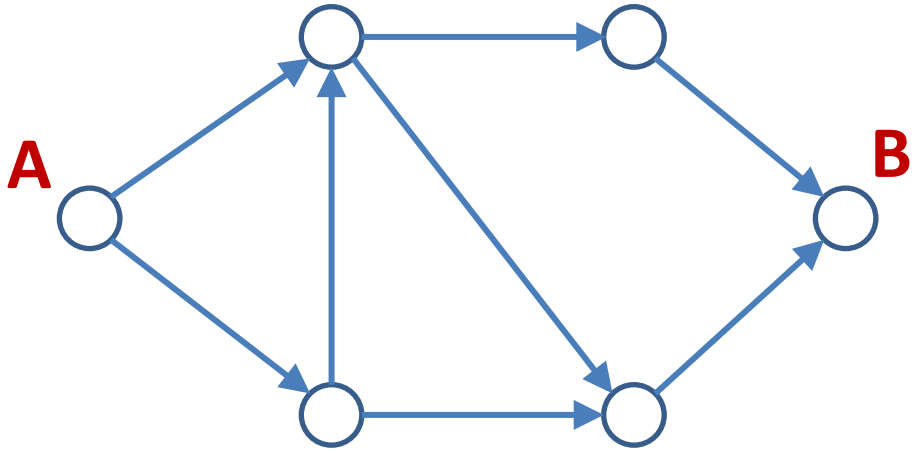
**Rule of Product :**

If one event can occur in m ways and another event can occur in n ways,

then there are $m \times n$ ways that both of these events can occur together

# Examples

- Consider the following map :




- How many different ways can we travel from A to B ?

# Examples

- For n given weights, what is the greatest number of different amount that can be made up by a combination of these weights ?
- We assume that weights can only be put on the same side of the balance

# Examples

- Answer : $2^n - 1$ (why?)
- Challenge :

  If Alison wants to weigh different items with weight equal to an integer between 1 and 100, can you help her to design a minimum set of weights ?

# Examples

- How many divisors does 1400 have ?
- Answer :

  Since $1400 = 2^3 \times 5^2 \times 7$,

  the number of divisors = $(3+1) \times (2+1) \times (1+1) = 24$

  Indeed, these are all the divisors :

  1, 2, 4, 5, 7, 8, 10, 14, 20, 25, 28, 35, 40, 50, 56,

  70, 100, 140, 175, 200, 280, 350, 700, 1400

# More Examples

- There are

  5 Chinese books, 7 English books, 10 French books

  How many ways to choose two books of different languages from them ?

- Answer : $5 \times 7 + 5 \times 10 + 7 \times 10 = 155$ ways

# More Examples

- A password in a computer system has the following requirements :
  1. The length is between 6 and 8 characters ;
  2. Each character can be an uppercase letter, a lowercase letter, or a digit
  3. Each password must contain at least one digit

- How many possible passwords are there ?

# Principle of Inclusion-Exclusion

- How many bit strings of length eight either start with 1 or end with 00?

- Answer :

  number of strings that start with 1 : $2^7 = 128$

  number of strings that end with 00 : $2^6 = 64$

  number of strings that start with 1 and end with 00 : $2^5 = 32$

  ---

  The desired number : $128 + 64 - 32 = 160$

# Principle of Inclusion-Exclusion

- In general, we have the following rule :

**Inclusion-Exclusion Principle (for two sets) :**

If one event can occur in m ways or n ways, and there are r ways that are common to the two different ways, then the number of ways an event can occur is $m + n - r$

# Example

- There are 350 applicants to a job, and
  (i) 220 with major in CS
  (ii) 147 with major in Business
  (iii) 51 with major in both CS and Business
- How many have major neither in CS nor Business ?

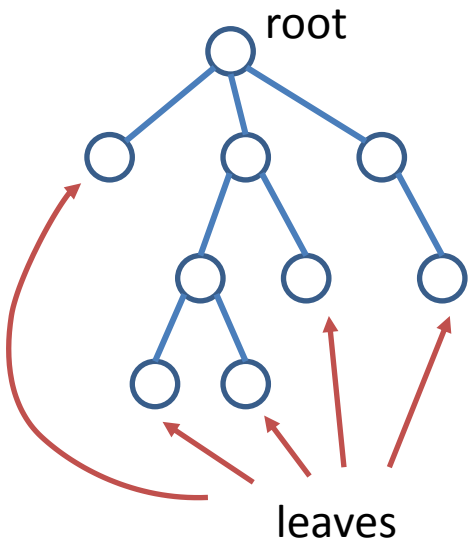
  # applicants with major in CS, Business, or both

  = 220 + 147 – 51 = 316

  ➔ The desired answer : 350 – 316 = 34

# Tree Diagrams

- A tree is a special type of graphs that contains
  (1) a root ;
  (2) branches leaving the root ;
  (3) possibly additional branches leaving the endpoints of other branches




- Nodes without any branches leaving are called leaves

# Tree Diagrams

- Counting problems can be solved using tree diagrams

- To do so, we use

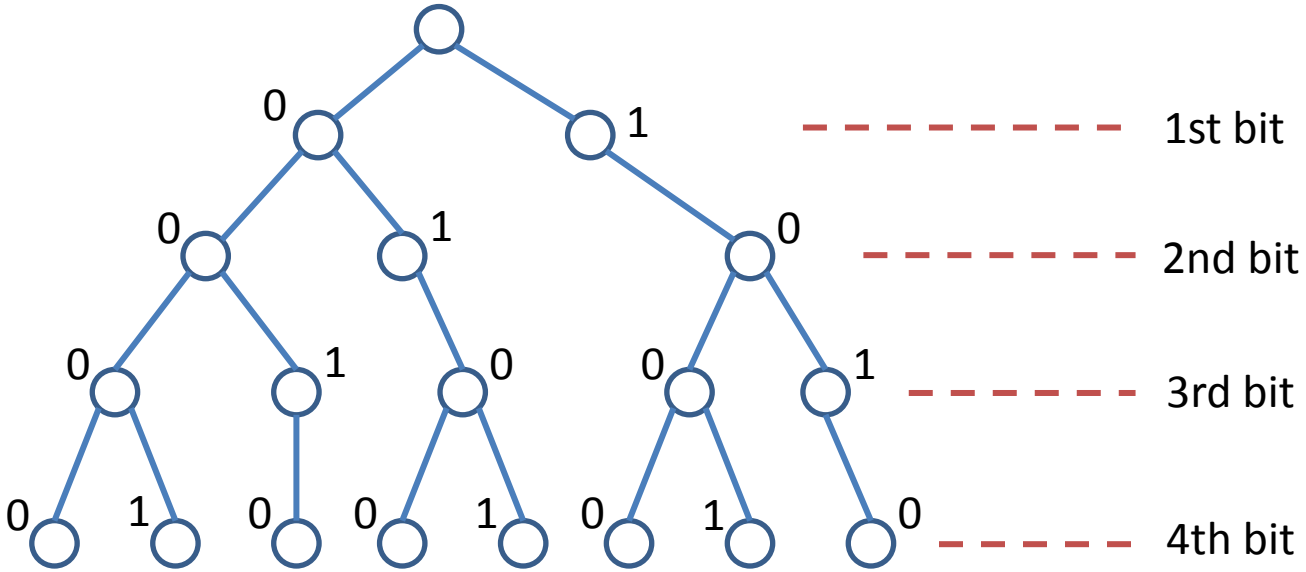
  (1) a branch to represent each possible choice, and

  (2) a leaf to represent each possible outcome

- Note : Number of choices of which branch to follow to reach a leaf can vary

# Examples

- How many bit strings of length four do not have two consecutive 1s ?



# Examples

- In a best-of-five playoff between two teams, whoever wins 3 games will win the playoff
- How many different ways can the playoff occur ?