Assignment 2: Solving Ax=b by Gaussian Elimination with Partial Pivoting

This exercise requires you to write an *efficient* and *effective* C/C++, Java, or Matlab program for solving a linear system of equations $A\mathbf{x} = \mathbf{b}$ by the strategy of LU decomposition with *partial pivoting.* Let n, $A \in R^{n\times n}$, $\mathbf{b} \in R^n$ be known, you are asked to write a module in C/C++, Java, or Matlab which takes n, A and $\mathbf{b}$ as input and store the output in $\mathbf{b}$. Test your program for the following problem. Other data sets given in *dataB.txt* may be used for your practice.

(1) Apply the Gaussian Elimination with Partial Pivoting method to solve the following equaitons.

$$\begin{array}{rcrcrcr} 4.00001x & + & 1.00000y & + & 2.00000z & = & 4.00001 \\ 10.00000x & - & 0.10000y & + & 3.00000z & = & 12.80000 \\ 5.00000x & + & 3.00000y & + & 1.00000z & = & 12.00000 \end{array}$$

(a) Write this equation as $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(b) Find $PA = LU$, where L is unit lower$-\Delta$ and U is upper$-\Delta$ (using either Matlab or C/C++/Java programs).

(c) Apply Gaussian elimination and back substitution to solve $PA\mathbf{x} = P\mathbf{b}$.

(d) Can you get the same solution without using the partial pivoting strategy for this problem?

(2) The $n \times n$ Hilbert matrix $H_n = [h_{ij}]$ is defined by

$$h_{ij} = \frac{1}{i+j-1}, \quad i, j, = 1, 2, \cdots, n$$

H_n can be generated using the MATLAB function **hilb**(n). It is well known that the Hilbert matrix is nortoriously ill-conditioned.

(a) Generate H_n, and $\mathbf{x}_n = \mathbf{1}_n = \mathbf{ones}(n, 1)$ for $n = 8, 12, 16, 20, 24$. In each case, construct $H_n\mathbf{x}_n = \mathbf{b}_n$ so that $\mathbf{1}_n$ is the exact solution. Then, by the given H_n and $\mathbf{b}_n$, applying the LU decomposition with partial pivoting strategy to solve $\mathbf{x}_n$.

(b) Find the determinant of each H_n in (**a**).

(c) Compute the condition number with p-norm, p=1,2,∞ for each H_n in (**a**).

(d) Discuss your solutions.