

# A Review of CS3331: Numerical Methods

Write matlab codes to solve the following problems.

1. Approximate  $\sin(x)$  by using Taylor expansion with the accuracy up to  $10^{-18}$  for  $x \in [-1, 1]$ .
2. Approximate  $\ln(1+x)$  by using Taylor expansion with the accuracy up to  $10^{-7}$  for  $x \in (0, 1)$ .
3. Apply *LU – decomposition* with partial pivoting and back substitution to solve the following linear system of equations.

$$\begin{aligned}x + \frac{1}{2}y + \frac{1}{3}z + \frac{1}{4}w &= 1 \\ \frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z + \frac{1}{5}w &= 1 \\ \frac{1}{3}x + \frac{1}{4}y + \frac{1}{5}z + \frac{1}{6}w &= 1 \\ \frac{1}{4}x + \frac{1}{5}y + \frac{1}{6}z + \frac{1}{7}w &= 1\end{aligned}$$

4. Find the characteristic polynomial of the matrix  $A$  given below and compare the roots of the characteristic equation of  $A$  with those obtained from the matlab command  $\text{eig}(A)$ .

$$A = \begin{bmatrix} -5 & 2 & 1 \\ 2 & -5 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

5. Solve the nonlinear systems of equations using Newton's method with a *suitable* initial guess.

$$\begin{aligned}x^2 + 20x + y^2 + z^2 &= 20 \\ x^2 + 20y + z^2 &= 20 \\ x^2 + y^2 - 40z &= 0\end{aligned}$$

6. Let  $X \sim \chi^2(4)$ , then  $f(x) = P(X = x) = \frac{1}{4}xe^{-x/2}$ ,  $x \geq 0$ . Try to find the median of  $X$ , that is,  $x^*$  such that  $\int_0^{x^*} f(t)dt = 0.5$ .