Project 2 - Data Visualization: PCA, LDA, Clustering

This project is designed for you to further understand PCA, LDA, and Data Clustering as introduced in class by real implementations on some data sets. Moreover, the implementation by Independent Component Analysis (ICA) is also *encouraged* but is *not required.*

(1) For the data set "8OX" introduced in class, there are $n = 45$ patterns from $k = 3$ categories, each pattern consists of $d = 8$ features. Each pattern can be denoted by $\mathbf{x}_i^{(k)}$, $1 \leq i \leq 15$, $1 \leq k \leq 3$, where $\mathbf{x}_i^{(k)} \in R^d$.

(a) Compute the pooled $d \times d$ covariance matrix $C = \frac{1}{n}\sum_{k=1}^{3}\sum_{i=1}^{n_k}(\mathbf{x}_i^{(k)} - \mathbf{u})(\mathbf{x}_i^{(k)} - \mathbf{u})^t$, where $\mathbf{u} = \frac{1}{n}\sum_{k=1}^{3}\sum_{i=1}^{15}\mathbf{x}_i^{(k)}$ is the mean vector.

(b) Report the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d$ of C.

(c) Report the percentage of $\gamma_j = \frac{\sum_{i=1}^{j}\lambda_i}{\sum_{i=1}^{d}\lambda_i}$, $\forall\ 1 \leq j \leq d$.

(d) Plot n patterns using the first *two* principal components.

(e) Plot n patterns using the first *three* principal components.

(f) Plot n patterns using the most *two* discriminative features based on linear discriminant analysis (LDA).

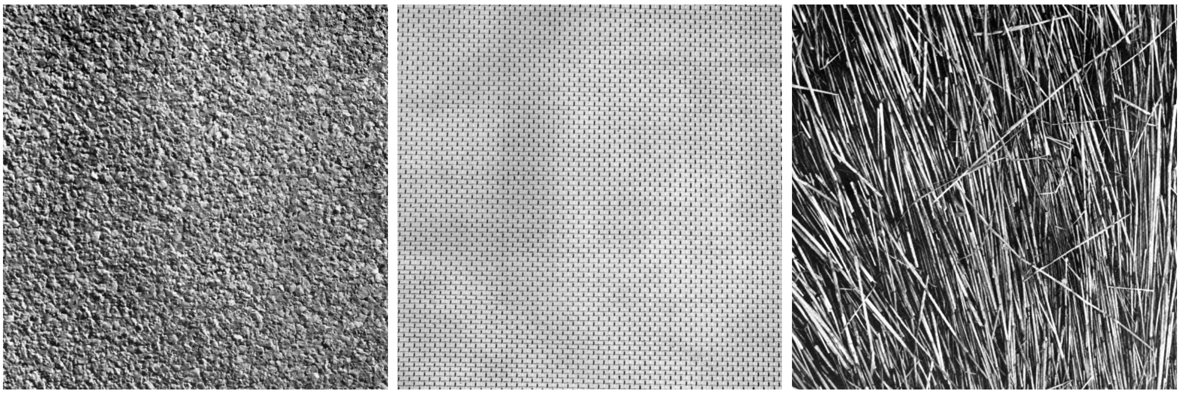
(g) Plot n patterns using the most *three* discriminative features based on linear discriminant analysis (LDA).

(h) Show the dendrogram (by complete linkage) of the original "8OX" data using the $d = 8$ features.

(i) Show the results of K-means implementation on the original "8OX" data using the $d = 8$ features.

(2) For the data set "data30x4" which were generated from two multivariate normal distributions with mean vector $\mathbf{u}_1 = [1, 1, 1, 1]^t$ and covariance matrix I, and mean vector $\mathbf{u}_2 = [-1, -1, 1, 1]^t$ and covariance matrix I, respectively, there are 15 patterns from $k = 2$ categories, each pattern consists of $d = 4$ features. Repeat the same processes as required in problem **(1)**.

(3) For the data set "53stdL3.txt" introduced in class, there are $n = 48$ patterns from $k = 3$ categories, each pattern consists of $d = 10$ features. The features are derived based on $(5/3) - wavelet\ transform$ on texture images as listed below. Repeat the same processes as required in problem **(1)**.



(a) (b) (c)

Figure 1: (a) D04, (b) D06, (c) D15 textures.