Solutions for CS3330: Fall 2020

14:20-15:30, December 9, 2020

1. Using $ln(1+x) \approx \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k} x^k + E_{n+1}(x)$ and find the minimum n such that $|\frac{(0.5)^n}{n} = \frac{1}{n2^n}| < 10^{-4}$, we have $n = 9$.

$$ln(1.5) \approx 0.5 - (0.25)/2 + 0.125/3 - 0.0625/4 + 0.03125/5 - 0.015625/6 + \cdots + \frac{1}{9 \times 512}$$

2(a) $\|\mathbf{x}\|_1 = 10, \quad \|\mathbf{x}\|_\infty = 4.$

2(b) $\|\mathbf{B}\|_1 = 34, \quad \|\mathbf{B}\|_\infty = 31.$

3.

$$H = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix}$$

4(a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 3 & 14 & 28 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix} = LU$$

4(b) Gaussian Elimination with partial pivoting and back substitution to solve $AX = \mathbf{b}$.

4(c) $X = [2, 1, -1]^t$.

5. The output array

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -1 & 3 & 3 \\ 2 & -3 & 1 \end{bmatrix}.$$

6. Lecture Notes or Referneces.

7. $\mathbf{x} = [2, 1]^t$.

8. All we have to do is to show that λ^k is an eigenvalue of A^k if λ is an eigenvalue of A. Suppose that $(\lambda, \mathbf{v})$ be eigenvalue-eigenvector pair of matrix A, then Since $A\mathbf{v} = \lambda\mathbf{v}$ implies $A^k\mathbf{v} = \lambda^k\mathbf{v}$ which completes the proof.