Subjects to Study in April

(1) Random Variables, Discrete vs. Continuous
(2) Probability Mass Function and Its Cumulative Distribution Function
(3) Examples
(4) Moment, Moment-Generating Function, Expectation, Variance
(5) Special Discrete Distributions (Bernoulli, Binomial, Poisson, etc.)
(6) Probability Density Function and Its Cumulative Distribution Function
(7) Examples
(8) Moment, Moment-Generating Function, Expectation, Variance
(9) Special Continuous Distributions (Exponential, Gamma, Normal, etc.)

$Discrete\ Moment - Generating\ Function$

- Let X be a discrete r.v. with probability mass function $p(x)\ on\ A$, then
- $E(X^n) = \sum_{x \in A} x^n p(x)$ is called the nth moment of X if it exists.
- When $n = 1, E(X)\ is\ the\ expected\ value\ or\ expectation$.
- When $n = 2, E(X^2)\ is\ called\ the\ second\ moment$.
- Note that $Var(X) = E(X^2) - (E[X])^2$
- $M(t) = E(e^{tX}) = \sum_{x \in A} e^{tx}\, p(x)$ is called the moment-generating function for X. Then $M^{(n)}(0) = E(X^n)$.
- Note that $M'(0) = E(X), and\ Var(X) = M''(0) - (M'(0))^2$

Moment-Generating Function for a Binomial Distribution

- If n Bernoulli trials all with probability of success *p* are performed independently, then X, the number of successes, is called a *binomial with parameters n and p*. The set of possible values of X is {0, 1, 2, …, n}. The probability mass function (pmf) is given as
- $f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \ \ if\ x = 0, 1, 2, \cdots, n$ (eq. 5.2)
- ~~$= 0\ \ elsewhere.$~~
- $M(t) = E(e^{tX}) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x (1-p)^{n-x} = (1 - p + pe^t)^n$
- $E(X) = np,\ Var(X) = np(1-p),\ \sigma_X = \sqrt{np(1-p)}$