Quiz 1, Spring 2020

due by 11:45, April 1, 2020

Name : _____ *SN* : _____ *Index* : ____

(10pts) 1. Solve the following problems by showing the brief procedures.

(a) A number is selected at random from the set $\{1, 2, \cdots, 63\}$. What is the probability that it is relatively prime to 63?

(b) A number is selected at random from the set $\{1, 2, \cdots, 1000\}$. What is the probability that it is neither divisible by 3 nor by 5?

(c) The coefficients of the quadratic equation $x^2 + bx + c = 0$ are determined by tossing a fair die twice (the first outcome is b, the second one is c). Find the probability that the equation has real roots.

(d) Find the number of *positive* integer solutions (x_1, x_2, x_3) for $x_1 + x_2 + x_3 = 8$.

(e) Calculate the coefficient of x^2y^3 in the binomial expansion of $(2x - 3y)^5$.

(3pts) 2. For any positive integer n, prove that $\begin{pmatrix} 2n \\ n \end{pmatrix} = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix}^2$.

(3pts) 3. Evaluate $\sum_{i=0}^{n}(-1)^i \begin{pmatrix} n \\ i \end{pmatrix} + \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix}$.

(3pts) 4. Suppose that 100 freshmen CS students took both Calculus and Physics in the same class, 70% passed Calculus, 60% passed Physics, and 50% passed both. If a randomly selected freshmen is found to have passed Calculus, what is the probability that he or she also passed Physics?

(3pts) 5. In a certain factory, machines A, B, C are producing masks of the same style and size. Of their production, machines A, B, C produce 2%, 1%, and 3% defective masks, respectively. Of the total production of masks, machine A produces 35%, machine B produces 25%, and machine C produces 40% masks, respectively. If one mask is randomly selected and is defective, calculate the probability that it is produced from machine A, machine B, and machine C, respectively.

(3pts) 6. Show your procedures to compute the following integrals.

(a) $\gamma = \int_0^\infty e^{-x^2/2} dx$.

(b) $\Gamma(\frac{1}{2})$, where $\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} dt$