Quiz 2, Spring 2020

Due by 12:00 noon on May 29, 2020

$Name:$ _____ $SN:$ _____ $Index:$ _____

(5 pts)(1) Let R be the bounded region between $y = x$ and $y = x^2$. A random point (X, Y) is selected from R.

(a) Find the joint probability density function of X and Y, $f(x, y)$.

(b) Calculate the marginal probability density function $f_X(x)$.

(c) Calculate $E(X)$.

(5 pts)(2) Let X and Y be independent exponential random variables both with mean 1. Let $W = max(X, Y)$.

(a) Find the distribution function $F_W(w)$ of W.

(b) Calculate $f_W(w)$.

(c) Calculate $E(W)$.

(5 pts)(3) Let X and Y be independent and identically distributed exponential random variables with mean 3. Prove that $X/(X+Y)$ is *uniform* over $(0,1)$, that is, $X/(X+Y) \sim U(0,1)$.

(5 pts)(4) Let X and Y be continuous random variables with the joint probability density function given by $f(x,y) = e^{-x(y+1)} \quad if \quad x > 0,\ 0 \leq y \leq e-1$, and $f(x,y) = 0,\ elsewhere.$

Calculate $E(X|Y=y)$.

(5 pts)(5) Let X and Y have the joint probability density function $f(x, y) = 1,\ if\ 0 \leq x,\ y \leq 1$.

(a) Calculate $P(X + Y \leq 3/4)$,

(b) Calculate $P(XY \leq 9/16)$,

(c) Calculate $P(Y \leq \sin(\pi X))$,