

Exam I for EECS3030(02), Spring 2020

10:10-11:45, May 6, 2020

Name : _____ SN : _____ Index : _____

(30pts) 1. Choose the *best* (unique) solution for each of the following problems.

()**(a)** A number is selected at random from the set $\{1, 2, \dots, 30\}$. What is the probability that it is relatively prime to 180?

(1) $\frac{2}{15}$, (2) $\frac{4}{15}$, (3) $\frac{6}{15}$, (4) $\frac{8}{15}$, (5) none

()**(b)** The coefficient of the quadratic equation $ax^2 - 2bx + c = 0$ are determined by tossing a fair die three times (the first outcome is a , the second outcome is b , and the third one is c). The probability that the equation has two distinct real roots is

(1) $\frac{8}{216}$, (2) $\frac{25}{216}$, (3) $\frac{173}{216}$, (4) $\frac{193}{216}$, (5) none

()**(c)** How many divisors does 750 have?

(1) 8, (2) 10, (3) 12, (4) 16, (5) none

()**(d)** The number of nonnegative integer solutions (x_1, x_2, x_3) for $x_1 + x_2 + x_3 = 5$ is

(1) 6, (2) 10, (3) 15, (4) 21, (5) none

()**(e)** The coefficient of x^2y^3 in the binomial expansion of $(3x - 2y)^5$ is

(1) -1080, (2) -720, (3) 720, (4) 1080, (5) none

()**(f)** $\sum_{k=1}^n \binom{n}{k} 2^k =$

(1) $3^n - 1$, (2) $2^n - 1$, (3) 3^n , (4) 2^n , (5) none

()**(g)** $\sum_{k=1}^n \binom{n}{k} (-1)^k =$

(1) -1, (2) 0, (3) 1, (4) 2^n , (5) none

()**(h)** A list of all permutations of 13579 is put in an increasing order. The 100th number in the list is

(1) 91537, (2) 91573, (3) 93157, (4) 93175, (5) none

()**(i)** A number is selected at random from the set $\{1, 2, \dots, 1000\}$. The probability that it is divisible neither by 3 nor by 5 is

(1) 0.60, (2) 0.50, (3) 0.40, (4) 0.30, (5) none

()**(j)** Let X be a randomly selected point from the interval $(0, 3)$. Then the probability that $P(X^2 - 3X + 2 > 0 \mid X)$ is

(1) $\frac{1}{3}$, (2) $\frac{2}{3}$, (3) $\frac{3}{4}$, (4) 1, (5) none

(10pts)2. Let the probability density function of X be $f(x) = \beta x^{\beta-1}$, $0 < x < 1$, $0 < \beta < \infty$. Show that $Y = -2\beta \ln(X)$ has an exponential distribution with $E(Y) = 2$.

(40pts)3. Fill the following blanks.

(a) Let A and B be independent events with $P(A) = 0.8$ and $P(B) = 0.6$, then

$$P(A \cap B) = \underline{\hspace{2cm}}, \quad P(A \cup B) = \underline{\hspace{2cm}}$$

(b) Let a binomial random variable X have the probability mass function $f(x) = \binom{50}{x} (0.8)^x (0.2)^{50-x}$, $0 \leq x \leq 50$. Then

$$E(X) = \underline{\hspace{2cm}}, \quad Var(X) = \underline{\hspace{2cm}}, \quad M(t) = \underline{\hspace{2cm}}$$

(c) If the moment-generating function of X is $M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$, then

$$E(X) = \underline{\hspace{2cm}}, \quad Var(X) = \underline{\hspace{2cm}}$$

(d) Let a random variable X have the Poisson distribution with variance 4. Then

$$E(X) = \underline{\hspace{2cm}}, \quad M(t) = \underline{\hspace{2cm}}$$

(e) Let a random variable X have the geometric distribution with $E(X) = 2$. Then

$$Var(X) = \underline{\hspace{2cm}}, \quad M(t) = \underline{\hspace{2cm}}$$

(f) A random variable X has p.d.f. $f(x) = \frac{1}{2}e^{-x/2}$, $x \geq 0$. Then

$$\text{the 25th percentile of } X = \underline{\hspace{2cm}}, \quad \text{the median of } X = \underline{\hspace{2cm}}$$

(g) Let $X \sim N(3, 1)$, then the p.d.f. of X is $f(x) = \underline{\hspace{2cm}}$,

$$\text{the moment-generating function } \phi_X(t) = \underline{\hspace{2cm}}$$

(h) Let a r.v. X have the probability density function $f(x) = \frac{1}{16}x^2e^{-x/2}$ $\forall x > 0$. Then

$$\text{The moment-generating function } \phi_X(t) = \underline{\hspace{2cm}}, \quad Var(X) = \underline{\hspace{2cm}}$$

(i) Let $Z \sim N(0, 1)$ be the standard normal distribution, define $Y = Z^2$. Then

$$\text{The moment-generating function } \phi_Y(t) = \underline{\hspace{2cm}}, \quad Var(Y) = \underline{\hspace{2cm}}$$

(j) Define $\Gamma(\alpha) = \int_0^\infty e^{-t}t^{\alpha-1}dt$, then $\Gamma(3) = \underline{\hspace{2cm}}, \quad \Gamma(1.5) = \underline{\hspace{2cm}}$

(10pts)4. Given a random permutation of the integers in the set $\{1, 2, 3, 4, 5\}$, let X be equal the number of integers that are in their natural position. Then the moment-generating function of X is

$$M(t) = \frac{44}{120} + \frac{45}{120}e^t + \frac{20}{120}e^{2t} + \frac{10}{120}e^{3t} + \frac{1}{120}e^{5t}$$

- (a) Find $E(X)$ and $Var(X)$.
- (b) Find the probability that at least one integer is in its natural position.
- (c) Draw a graph of the histogram of the probability mass function of X .

(10pts)5. Suppose that 80% of the seniors, 70% of the juniors, 50% of the sophomores, and 30% of the freshmen of a college use the library of their campus frequently. If 30% of all students are freshmen, 25% are sophomores, 25% are juniors, and 20% are seniors.

- (a) What is the probability that a student uses the library frequently?
- (b) If a student uses the library frequently, what is the probability that he or she is a sophomore?
- (c) If a student uses the library frequently, what is the probability that he or she is a junior.