

H4: Vector Quantization by LBG Algorithm

LBG algorithm is like a K-means clustering algorithm which takes a set of input vectors $S = \{\mathbf{x}_i \in R^d \mid i = 1, 2, \dots, n\}$ as input and generates a representative subset of vectors $C = \{\mathbf{c}_j \in R^d \mid j = 1, 2, \dots, K\}$ with a user specified $K \ll n$ as output according to the similarity measure. For the application of Vector Quantization (VQ), $d = 16$, $K = 256$ or 512 are commonly used.

LBG Algorithm

1. Input training vectors $S = \{\mathbf{x}_i \in R^d \mid i = 1, 2, \dots, n\}$.
2. Initiate a codebook $C = \{\mathbf{c}_j \in R^d \mid j = 1, 2, \dots, K\}$.
3. Set $D_0 = 0$ and let $k = 0$.
4. Classify the n training vectors into K clusters according to $\mathbf{x}_i \in S_q$ if $\|\mathbf{x}_i - \mathbf{c}_q\|_p \leq \|\mathbf{x}_i - \mathbf{c}_j\|_p$ for $j \neq q$.
5. Update cluster centers \mathbf{c}_j , $j = 1, 2, \dots, K$ by $\mathbf{c}_j = \frac{1}{|S_j|} \sum_{\mathbf{x}_i \in S_j} \mathbf{x}_i$.
6. Set $k \leftarrow k + 1$ and compute the distortion $D_k = \sum_{j=1}^K \sum_{\mathbf{x}_i \in S_j} \|\mathbf{x}_i - \mathbf{c}_j\|_p$.
7. If $(D_{k-1} - D_k)/D_k > \epsilon$ (*a small number*), repeat steps 4 ~ 6.
8. Output the codebook $C = \{\mathbf{c}_j \in R^d \mid j = 1, 2, \dots, K\}$,

The convergence of LBG algorithm depends on the initial codebook C , the distortion D_k , and the threshold ϵ , in implementation, we need to provide a maximum number of iterations to guarantee the convergence.

This homework asks you to take $4 \sim 6$ *nrows* \times *ncols* (e.g., *nrows*=*ncols*=512) gray level images as input for LBG algorithm to train a codebook of size 256 and 512, respectively. Then perform image compression by VQ and show the PSNR values and CPU times for the images in the training set and not in the training set.